CHAPTER FIVE
Surface Runoff

5.1 INTRODUCTION

Rainfall excess is that portion of total rainfall that is not stored on the land surface or infiltrated into underlying soil. It eventually comprises direct runoff to downstream rivers, streams, storm sewers, and other conveyance systems. One of the key parameters in the design and analysis of urban hydrologic systems is the resulting peak runoff or, in some cases, the variation of runoff over time (i.e., hydrograph) at a watershed outlet or other downstream design point. Its evaluation requires an adequate understanding of the processes and routes by which the transformation of excess rainfall to direct runoff occurs.

It is worth noting that the models described herein for runoff estimation are characterized as lumped methods. In this respect, they use a single set of characteristic parameters to describe an entire basin. For example, the unit hydrograph methods assume rainfall excess to be uniform across a watershed, capable of being described by a single hyetograph. Strictly speaking, such parameters vary spatially; however, it is generally not feasible to account for the variability (e.g., runoff from every lawn, street, or roof), as in the case of a distributed model.

5.2 TIME OF CONCENTRATION

Various parameters are used to characterize the response of a watershed to a rainfall event. Of these, the most frequently used is time of concentration, \( t_c \), defined as the time required for water to travel from the most hydraulically-remote portion of a watershed to a location of interest (e.g., basin outlet). This also corresponds to the response time from the beginning of a storm event to a time when the entire basin contributes runoff to that location. Beyond the time of concentration, runoff will remain constant until rainfall excess ceases.

Because \( t_c \) cannot be directly measured, it is often estimated based on travel times along appropriately partitioned flow paths that include both overland flow and more concentrated channel flow components. Flow times can depend on physiographic factors such as watershed size, topography, and land use. Climatic factors, such as rainfall intensity and duration, play an equally important role. For urban drainage systems, the timing of runoff may also be heavily influenced by a storm water collection system. In this case, time of concentration should be adjusted to reflect the additional travel time through the conveyance system.
Numerous empirical and physically-based methods are available for approximating time of concentration. A study conducted by McCuen et al. (1984), however, compared these various methods and showed that results can vary significantly. Thus, the choice of which method to apply is not an easy one for the analyst. The selection requires careful consideration of limitations of each method and the conditions under which it was derived. In addition, it is often recommended to estimate $t_c$ using several methods and use qualitative judgment to arrive at a single value. Note that in practice, a minimum value of five minutes is recommended for urban applications and ten minutes for residential applications.

5.2.1 Overland Flow Models

Overland flow is that portion of runoff that occurs as sheet flow over a land surface without becoming concentrated in well-defined channels, gullies, and rills. A common example is flow over long, gradually-sloped pavements during or immediately following a storm. Some of the methods used for computing $t_c$ for overland flow include the kinematic wave model, NRCS models, and a variety of empirical techniques.

5.2.1.1 Kinematic Wave Model

Kinematic wave theory relies on the continuity equation (i.e., conservation of mass) and a simplified form of the momentum equation to derive solutions for flow problems. For a unit width of overland flow, the former can be expressed as

$$\frac{\partial q}{\partial x} + \frac{\partial y}{\partial t} = i$$

where $q$ is the unit width flow rate; $x$ is longitudinal distance along the flow path; $y$ is flow depth; $t$ is time; and $i$ is the rate of rainfall, or rainfall excess in the case that abstractions are considered. Note that the two terms on the left-hand side of Equation 5-1 are used to simulate the non-uniform and unsteady flow aspects (i.e., spatial and temporal variation of flow), respectively. With respect to momentum, however, flow is assumed to be steady and uniform from one time increment to the next. As described in greater detail in Chapter 6, the implication is that simulated kinematic waves will not appreciably accelerate and can only flow in the downstream direction. Thus, a wave will be observed as relatively uniform rise and fall in the water surface over a long period. The method is, therefore, limited to conditions that do not demonstrate appreciable attenuation.
Consider that, for uniform flow, the momentum equation can be expressed in the general form

\[ q = \alpha_k y^m \]  \hspace{1cm} (5-2)

where \( \alpha_k \) and \( m \) are constants that depend on a relationship between depth and discharge, \( Q \). For example, the Manning equation represents one such relationship and can be expressed as

\[ Q = \frac{K_m}{n} A R^{2/3} S^{1/2} \]  \hspace{1cm} (5-3)

where \( K_m \) is a constant equal to 1.49 in U.S. customary units and 1.0 in S.I. units; \( n \) is the Manning roughness coefficient; \( A \) is effective flow area; \( R \) is the hydraulic radius, defined as the ratio of flow area to wetted perimeter, \( P \); and \( S \) is surface slope in ft/ft or m/m. Since overland flow can be considered as shallow flow through a very wide rectangular channel, hydraulic radius can be approximated as depth, and Equation 5-3 can be rewritten as

\[ Q = \frac{K_m}{n} B y^{2/3} S^{1/2} \]  \hspace{1cm} (5-4)

where \( B \) is flow width. In addition, to be consistent with the current kinematic wave analysis, Equation 5-4 can be expressed for a unit width as

\[ q = \frac{K_m}{n} y^{5/3} S^{1/2} \]  \hspace{1cm} (5-5)

Comparing Equations 5-5 and 5-2, \( \alpha_k \) can be taken as

\[ \alpha_k = \frac{K_m}{n} S^{1/2} \]  \hspace{1cm} (5-6)

and \( m \) is 5/3. Combining these two expressions yields the kinematic wave equation for overland flow.
where \( c_k \) is the kinematic wave celerity (i.e., speed), equal to

\[
c_k = \alpha_k m y^{m-1}
\]  

(5-8)

Equation 5-7 implies that to an observer moving at a speed (i.e., \( \frac{dx}{dt} \)) equivalent to \( c_k \), the relationship between depth of flow and rainfall excess is

\[
\frac{dy}{dt} = i
\]  

(5-9)

Solving this relationship at initial conditions \( y = 0 \) everywhere at \( t = 0 \) yields

\[
y = it
\]  

(5-10)

Substituting this result into Equation 5-8 and integrating subject to the boundary condition \( x = 0 \) at \( t = 0 \) gives

\[
x = \alpha_k t^{m-1} i^m
\]  

(5-11)

This expression can be used to evaluate the time required for a kinematic wave to travel an overland flow path of distance \( L \), which is assumed to be equal to the time of concentration. The corresponding general relationship is

\[
t_c = \left( \frac{L}{\alpha_k i^{m-1}} \right)^{1/m}
\]  

(5-12)

Using the Manning equation to relate depth and discharge (i.e., \( m = 5/3 \) and \( \alpha_k \) by Equation 5-6) (Morgali and Linsley, 1965; Aron and Erborge, 1973),

\[
t_c = \frac{0.938 L^{0.6} n^{0.6}}{i^{0.4} S^{0.3}}
\]  

(5-13)
where $t_c$ is in minutes; $L$ is ft; $n$ can be read from Table 5-1; $i$ is in in/hr and is assumed to be uniform over the catchment; and $S$ is in ft/ft. Note that because the slope is considered to be constant over $L$, the time of concentration should be computed and summed over relatively small topographic contour intervals.

Table 5-1: Manning roughness coefficients for overland flow surfaces

<table>
<thead>
<tr>
<th>Surface description</th>
<th>Manning $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete, asphalt</td>
<td>0.010 – 0.016</td>
</tr>
<tr>
<td>Bare sand</td>
<td>0.010 – 0.016</td>
</tr>
<tr>
<td>Gravel</td>
<td>0.012 – 0.030</td>
</tr>
<tr>
<td>Bare clay-loam (eroded)</td>
<td>0.012 – 0.033</td>
</tr>
<tr>
<td>Natural rangeland</td>
<td>0.010 – 0.320</td>
</tr>
<tr>
<td>Bluegrass sod</td>
<td>0.39 – 0.63</td>
</tr>
<tr>
<td>Short-grass prairie</td>
<td>0.10 – 0.20</td>
</tr>
<tr>
<td>Dense grass, Bermuda grass, bluegrass</td>
<td>0.17 – 0.48</td>
</tr>
<tr>
<td>Forestland</td>
<td>0.20 – 0.80</td>
</tr>
</tbody>
</table>

Since the time of concentration and rainfall intensity are both unknown, application of Equation 5-13 is iterative. An initial intensity is assumed and the corresponding value of $t_c$ is computed. The assumed value of $i$ must be checked by determining a new time of concentration based on intensity-duration-frequency (IDF) relationships and comparing the new value with that previously computed. This process is repeated until values for intensity in successive iterations converge. Overton and Meadows (1976) used a power law to relate rainfall intensity and duration in order to bypass the need for this iterative solution. By substituting the 2-year, 24-hour rainfall depth, $P_{24}$, for $i$, they proposed

$$t_c = \frac{0.42 L^{0.8} n^{0.8}}{P_{24}^{0.5} S^{0.4}} \quad (5-14)$$

where $P_{24}$ is in inches and can be obtained from corresponding IDF data.

Since the kinematic wave model represented by Equations 5-13 and 5-14 was derived using Manning equation, it is inherently limited to turbulent flows. Furthermore, the method assumes that no local inflow occurs; no backwater or storage effects are present; the discharge varies only with depth; and that
planar, non-converging flow predominates. Unfortunately, these assumptions become less realistic as land slope decreases, surface roughness increases, or the length of flow path increases (McCuen, 1998). Practical upper limits on length range from 100 to 300 ft (30 to 90 m).

5.2.1.2 NRCS Methods

The Natural Resources Conservation Service (NRCS), formerly the Soil Conservation Service (SCS), has recommended two alternative methods for estimating time of concentration. The first is based on a relationship for basin lag time, defined as the time between the center of mass of rainfall and peak discharge of the corresponding hydrograph. The agency estimated that time of concentration is typically \( \frac{5}{3} \) of the lag time. Combining these two relationships yields (SCS, 1986)

\[
\frac{L^{0.3}(1000/CN) - 9^{0.7}}{190S^{0.2}}
\]

where the curve number, \( CN \), can be obtained from Chapter 3. For a highly urban area, the time of concentration can be multiplied by an adjustment factor, \( M \), determined by

\[
M = 1 - p \left( -6.8 \times 10^{-3} + 3.4 \times 10^{-4} CN - 4.3 \times 10^{-7} CN^2 - 2.2 \times 10^{-8} CN^3 \right)
\]

where \( p \) is the percent imperviousness or percent of main channels that are hydraulically improved beyond natural conditions. The NRCS recommends that Equation 5-15 be limited to homogeneous watersheds that have curve numbers between 50 and 95 and that are smaller than 2,000 acres (800 ha).

The second method recommended by the NRCS is the upland method, which can be expressed as

\[
t_c = \frac{L}{60V}
\]

where \( V \) is the velocity of overland flow in fps or m/s, which can be estimated from Figure 5-1 based on land use and surface slope in percent. For cases where the primary flow path can be divided into various segments having different slopes or land uses, \( t_c \) should be evaluated by
\[ t_c = \frac{1}{60} \sum_{j=1}^{N} \left( \frac{L_j}{V_j} \right) \]  

(5-18)

where \( j \) refers to a particular flow segment.

Figure 5-1: Velocities for use in the upland method (SCS, 1986)

An alternative to using Figure 5-1 is to express velocity as

\[ V = K \sqrt{S} \]  

(5-19)
where \( V \) is in fps; \( K \) is conveyance, listed in Table 5-2 for various surfaces; and \( S \) is in ft/ft. The basis for these values, as well as the basis for Figure 5-1, are various assumed combinations of flow geometry and surface roughness.

**Table 5-2: Conveyance values for overland flow**

<table>
<thead>
<tr>
<th>Surface description</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forestland</td>
<td>0.7 – 2.5</td>
</tr>
<tr>
<td>Grass</td>
<td>1.0 – 2.1</td>
</tr>
<tr>
<td>Short-grass prairie</td>
<td>7.0</td>
</tr>
<tr>
<td>Natural rangeland</td>
<td>1.3</td>
</tr>
<tr>
<td>Paved area</td>
<td>20.4</td>
</tr>
</tbody>
</table>

Source: Adapted from McCuen (1998)

Care should be taken when applying Equation 5-19 for slopes that exceed approximately four percent. In these instances, velocity profiles become more complex and \( V \) tends to be overestimated.

### 5.2.1.3 Kirpich Equation

The Kirpich equation for time of concentration can be expressed as (Kirpich, 1940)

\[
    t_c = 0.0078 \frac{L^{0.77}}{S^{0.385}}
\]

This relationship was originally developed from SCS data for well-defined and relatively steep channels draining small- to moderate-sized watersheds (i.e., < 100 acres), but it often yields satisfactory results for overland flow on bare soils and for areas up to 200 acres (80 ha). Note that for more general cases of overland flow, Rossmiller (1980) recommends that \( t_c \) be multiplied by an adjustment factor 2.0. For concrete or asphalt surfaces, the adjustment factor reduces to 0.4.

### 5.2.1.4 Izzard Equation

Based on a series of laboratory experiments by the Bureau of Public Roads, Izzard (1946) proposed the following relationship for time of concentration for roadway and turf surfaces:
\[ t_c = \frac{41.025(0.0007i + c)L^{1/3}}{S^{1/3}t^{1/3}} \]  

(5-21)

where \( c \) is a retardance factor that ranges from 0.007 for smooth pavement to 0.012 for concrete and to 0.06 for dense turf. The method is designed for applications in which the product of intensity (in/hr) and flow length (ft) is less than 500. In addition, application of 5-21 requires an iterative solution, similar to that of the kinematic wave model, since \( i \) is dependent on time of concentration.

### 5.2.1.5 Kerby Equation

Kerby (1959) defined flow length as the straight-line distance from the most distant point of a basin to its outlet, measured parallel to the surface slope. Based on this definition, time of concentration can be evaluated as

\[ t_c = \frac{0.83(nL)^{0.47}}{S^{1/3}} \]  

(5-22)

This relationship is not commonly used and has the most limitations. It was developed based on watersheds less than 10 acres (4 ha) in size and having slopes less than one percent. It is generally applicable for flow lengths less than 1,000 ft (300 m).

### 5.2.1.6 FAA Method

The Federal Aviation Administration (FAA, 1970) used airfield drainage data assembled by the U.S. Army Corps of Engineers to develop an estimate for time of concentration. The method has been widely used for overland flow in urban areas and can be expressed as

\[ t_c = \frac{0.39(1.1 - C)L^{1/2}}{S^{1/3}} \]  

(5-23)

where \( C \) is a dimensionless runoff coefficient, which can be obtained from Table 5-3.
### Table 5-3: Runoff coefficients for 2 to 10 year return periods

<table>
<thead>
<tr>
<th>Description of drainage area</th>
<th>Runoff coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Business</strong></td>
<td></td>
</tr>
<tr>
<td>Downtown</td>
<td>0.70 - 0.95</td>
</tr>
<tr>
<td>Neighborhood</td>
<td>0.50 - 0.70</td>
</tr>
<tr>
<td><strong>Residential</strong></td>
<td></td>
</tr>
<tr>
<td>Single-family</td>
<td>0.30 - 0.50</td>
</tr>
<tr>
<td>Multi-unit detached</td>
<td>0.40 - 0.60</td>
</tr>
<tr>
<td>Multi-unit attached</td>
<td>0.60 - 0.75</td>
</tr>
<tr>
<td>Suburban</td>
<td>0.25 - 0.40</td>
</tr>
<tr>
<td>Apartment dwelling</td>
<td>0.50 - 0.70</td>
</tr>
<tr>
<td><strong>Industrial</strong></td>
<td></td>
</tr>
<tr>
<td>Light</td>
<td>0.50 - 0.80</td>
</tr>
<tr>
<td>Heavy</td>
<td>0.60 - 0.90</td>
</tr>
<tr>
<td>Parks and cemeteries</td>
<td>0.10 - 0.25</td>
</tr>
<tr>
<td>Railroad yards</td>
<td>0.20 - 0.35</td>
</tr>
<tr>
<td>Unimproved areas</td>
<td>0.10 - 0.30</td>
</tr>
<tr>
<td><strong>Pavement</strong></td>
<td></td>
</tr>
<tr>
<td>Asphalt</td>
<td>0.70 - 0.95</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.80 - 0.95</td>
</tr>
<tr>
<td>Brick</td>
<td>0.75 - 0.85</td>
</tr>
<tr>
<td><strong>Roofs</strong></td>
<td>0.75 - 0.95</td>
</tr>
<tr>
<td><strong>Lawns</strong></td>
<td></td>
</tr>
<tr>
<td>Sandy soils</td>
<td></td>
</tr>
<tr>
<td>Flat (2%)</td>
<td>0.05 - 0.10</td>
</tr>
<tr>
<td>Average (2 – 7 %)</td>
<td>0.10 - 0.15</td>
</tr>
<tr>
<td>Steep ( ≥ 7%)</td>
<td>0.15 - 0.20</td>
</tr>
<tr>
<td>Heavy soils</td>
<td></td>
</tr>
<tr>
<td>Flat (2%)</td>
<td>0.13 - 0.17</td>
</tr>
<tr>
<td>Average (2 – 7 %)</td>
<td>0.18 - 0.22</td>
</tr>
<tr>
<td>Steep ( ≥ 7%)</td>
<td>0.25 - 0.35</td>
</tr>
</tbody>
</table>

Source: Adapted from ASCE (1992)

5.2.1.7 Yen and Chow Method

Yen and Chow (1983) proposed the following expression for evaluation of time of concentration:
\[ t_c = K_f \left( \frac{NL}{S^{0.5}} \right)^{0.6} \]  

(5-24)

where \( K_f \) ranges from 1.5 for light rain \( (i < 0.8 \text{ in/hr}) \) to 1.1 for moderate rain \( (0.8 < i < 1.2 \text{ in/hr}) \), and to 0.7 for heavy rain \( (i > 1.2 \text{ in/hr}) \); and \( N \) is an overland texture factor, listed in Table 5-4.

**Table 5-4: Overland texture factor \( N \)**

<table>
<thead>
<tr>
<th>Overland flow surface</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth asphalt pavement</td>
<td>0.010</td>
<td>0.012</td>
<td>0.015</td>
</tr>
<tr>
<td>Smooth impervious surface</td>
<td>0.011</td>
<td>0.013</td>
<td>0.015</td>
</tr>
<tr>
<td>Tar and sand pavement</td>
<td>0.012</td>
<td>0.014</td>
<td>0.016</td>
</tr>
<tr>
<td>Concrete pavement</td>
<td>0.014</td>
<td>0.017</td>
<td>0.020</td>
</tr>
<tr>
<td>Rough impervious surface</td>
<td>0.015</td>
<td>0.019</td>
<td>0.023</td>
</tr>
<tr>
<td>Smooth bare packed soil</td>
<td>0.017</td>
<td>0.021</td>
<td>0.025</td>
</tr>
<tr>
<td>Moderate bare packed soil</td>
<td>0.025</td>
<td>0.030</td>
<td>0.035</td>
</tr>
<tr>
<td>Rough bare packed soil</td>
<td>0.032</td>
<td>0.038</td>
<td>0.045</td>
</tr>
<tr>
<td>Gravel soil</td>
<td>0.025</td>
<td>0.032</td>
<td>0.045</td>
</tr>
<tr>
<td>Mowed poor grass</td>
<td>0.030</td>
<td>0.038</td>
<td>0.045</td>
</tr>
<tr>
<td>Average grass, closely clipped sod</td>
<td>0.040</td>
<td>0.050</td>
<td>0.060</td>
</tr>
<tr>
<td>Pasture</td>
<td>0.040</td>
<td>0.055</td>
<td>0.070</td>
</tr>
<tr>
<td>Timberland</td>
<td>0.060</td>
<td>0.090</td>
<td>0.120</td>
</tr>
<tr>
<td>Dense grass</td>
<td>0.060</td>
<td>0.090</td>
<td>0.120</td>
</tr>
<tr>
<td>Shrubs and bushes</td>
<td>0.080</td>
<td>0.120</td>
<td>0.180</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Land use</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business</td>
<td>0.014</td>
<td>0.022</td>
<td>0.035</td>
</tr>
<tr>
<td>Semi-business</td>
<td>0.022</td>
<td>0.035</td>
<td>0.050</td>
</tr>
<tr>
<td>Industrial</td>
<td>0.020</td>
<td>0.035</td>
<td>0.050</td>
</tr>
<tr>
<td>Dense residential</td>
<td>0.025</td>
<td>0.040</td>
<td>0.060</td>
</tr>
<tr>
<td>Suburban residential</td>
<td>0.030</td>
<td>0.055</td>
<td>0.080</td>
</tr>
<tr>
<td>Parks and lawns</td>
<td>0.040</td>
<td>0.075</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Source: Yen and Chow (1983)
5.2.2 Channel Models

Overland flow eventually drains into defined channels, including street gutters, swales, storm sewers, and streams, and corresponding methods for computing time of concentration should be modified accordingly. For example, the Kirpich equation is applicable to channel flow by omitting the adjustment factor for overland flow. If, however, the channel is comprised of concrete or natural grass, Rossmiller (1980) recommends that adjustment factors of 0.2 and 2.0, respectively, be applied. Likewise, the NRCS upland method can be modified by letting conveyance, \( K \), in Equation 5-19 be

\[
K = \frac{K_m R^{2/3}}{n} \tag{5-25}
\]

where \( R \) can be evaluated for a particular channel using the formulas listed in Table 5-5, and \( n \) can be read from Table 5-6. By computing conveyance in this manner, \( V \) in Equation 5-19 is essentially being evaluated via the Manning equation for steady, uniform flow. In addition to these techniques, several empirical methods have been developed for channel- and mixed-flow conditions.

5.2.2.1 Van Sickle Equation

The Van Sickle equation was developed based on data from urban watersheds having areas less than 36 mi\(^2\) (93 km\(^2\)). The relationship can be expressed as (McCuen, 1998)

\[
t_c = 0.55 \left( \frac{L_t L_m}{S^{1/2}} \right)^{0.13} \tag{5-26}
\]

where \( L_t \) is the length in miles of all channels and storm sewers having a diameter greater than 3 ft (1 m), and \( L_m \) is the total length of the basin in miles.

5.2.2.2 Eagleson Equation

Eagleson (1962) derived the following expression for time of concentration using data from basins having areas less than 8 mi\(^2\) (21 km\(^2\)):

\[
t_c = 0.0111 \frac{L_m}{R^{2/3} S^{1/2}} \tag{5-27}
\]
where \( S \) and \( n \) are for the main collection sewer; and \( R \) is for the main channel when flowing full.

### Table 5-5: Common geometric properties of channel sections

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area ((A))</th>
<th>Wetted Perimeter ((P))</th>
<th>Hydraulic radius ((R))</th>
<th>Top width ((B))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td></td>
<td>(By)</td>
<td>(B + 2y)</td>
<td>(By)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(B + 2y)</td>
<td>(b)</td>
</tr>
<tr>
<td>Trapezoidal</td>
<td>((b + zy)y)</td>
<td>(b + 2y\sqrt{1 + z^2})</td>
<td>(\frac{(b + zy)y}{b + 2y\sqrt{1 + z^2}})</td>
<td>(b + 2zy)</td>
</tr>
<tr>
<td>Parabolic</td>
<td>(\frac{2}{3}By)</td>
<td>(\frac{B}{2}\sqrt{1 + x^2} + \left(x \cdot \frac{1}{l}\right))</td>
<td>(\frac{4y}{3}\left[\sqrt{1 + x^2} + \left(x \cdot \frac{1}{l}\right)\right]^{-l})</td>
<td>(b\sqrt{\frac{y}{l}})</td>
</tr>
<tr>
<td>Circular</td>
<td>((\theta - \sin \theta)D^2)</td>
<td>(\theta D)</td>
<td>(\frac{D}{4}\left(1 - \frac{\sin \theta}{\theta}\right))</td>
<td>(D \sin \frac{\theta}{2}) or (2\sqrt{y(D - y)})</td>
</tr>
</tbody>
</table>

\( ^a \) For triangular sections, use \( b = 0 \)

\( ^b \) \( x = 4y/B \)

\( ^c \) use \( \theta \) in radians, where \( \theta = 2\cos^{-1}\left(l - \frac{2y}{D}\right) \)

Source: Adapted from Chow (1959)

### 5.2.2.3 Carter Equation

Time of concentration for basins comprised of natural channels and partially sewer ed land uses can be evaluated by the Carter (1961) equation, expressed as
\[ L_e = 100 \frac{L_m^{0.6}}{S_m^{0.3}} \]  

where \( L_m \) and \( S_m \) are flow length in miles and surface slope in ft/mi, respectively.

### Table 5-6: Typical Manning roughness coefficients

<table>
<thead>
<tr>
<th>Channel or conduit description</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass, glass</td>
<td>0.010</td>
</tr>
<tr>
<td>PVC</td>
<td>0.009</td>
</tr>
<tr>
<td>Welded steel</td>
<td>0.012</td>
</tr>
<tr>
<td>Riveted steel</td>
<td>0.015</td>
</tr>
<tr>
<td>Cast iron</td>
<td>0.013</td>
</tr>
<tr>
<td>Galvanized iron</td>
<td>0.016</td>
</tr>
<tr>
<td>Corrugated metal</td>
<td>0.024</td>
</tr>
<tr>
<td>Concrete, Asphalt</td>
<td>0.013</td>
</tr>
<tr>
<td>Wood stave</td>
<td>0.012</td>
</tr>
<tr>
<td>Brick</td>
<td>0.015</td>
</tr>
<tr>
<td>Clay drainage tile</td>
<td>0.013</td>
</tr>
<tr>
<td>Gravel</td>
<td>0.023</td>
</tr>
<tr>
<td>Smooth earth</td>
<td>0.018 - 0.40</td>
</tr>
<tr>
<td>Natural channels, streams</td>
<td>0.025 - 0.08</td>
</tr>
</tbody>
</table>

### 5.3 PEAK-FLOW MODELS

Methods used to predict quantity of runoff are broadly categorized as either peak-flow or continuous-flow models. Both models will be described in this chapter. Continuous flow models estimate the variation of runoff over time. Peak flow models estimate only peak runoff values, which is typically sufficient for the design of many storm water conveyance systems. While several models exist, the rational method is by far the most common method for estimating peak runoff in urban applications.
5.3.1 Rational Method

The rational method is based on the principle that the maximum rate of runoff from a drainage basin occurs when all parts of the watershed contribute to flow and that rainfall is distributed uniformly over the catchment area.

The rational method is based on the principle that the maximum rate of runoff from a drainage basin occurs when all parts of the watershed contribute to flow and that rainfall is distributed uniformly over the catchment area. Since it neglects temporal flow variation and routing of flow through a watershed, storm water collection system, and any storage facilities, the rational method should be used only for applications in which accuracy of runoff values is not essential. The empirical rational formula can be expressed as

\[ Q_p = \frac{CA}{K_R} \]  

(5-29)

where \( Q_p \) is peak runoff rate in cfs or m³/s; \( C \) is the dimensionless runoff coefficient, listed in Table 5-3, that is used to adjust for rainfall abstractions; \( i \) is rainfall intensity in in/hr or mm/hr for a duration equal to the time of concentration; \( A \) is the drainage area in acres or hectares; and \( K_R \) is a conversion constant equal to 1.0 in U.S. customary units and 360 in S.I. units.

For non-homogeneous drainage areas having variable land uses, a composite runoff coefficient, \( C_c \), should be used in the rational formula. The composite coefficient is expressed as

\[ C_c = \frac{\sum_{j=1}^{n} C_j A_j}{\sum_{j=1}^{n} A_j} \]  

(5-30)

where \( A_j \) is the area for land use \( j \); \( C_j \) is the dimensionless runoff coefficient for area \( j \); and \( n \) is the total number of land covers. If Equation 5-30 is substituted into Equation 5-29, the rational formula can be rewritten as

\[ Q_p = \frac{i \sum_{j=1}^{n} C_j A_j}{K_R} \]  

(5-31)

Application of the rational method is valid for drainage areas less than 200 ac (80 ha), which typically have times of concentration of less than 20 minutes (ASCE, 1992).
5.3.2 NRCS TR-55 Method

The NRCS proposed the TR-55 method for estimating peak runoff using the results from numerous small and mid-sized basins. The method is recommended for homogeneous catchments in which curve numbers (see Chapter 3) are greater than 50. The TR-55 method describes peak runoff as

$$Q_p = q_u A P_e F_p$$  \hspace{1cm} (5-32)

where $Q_p$ is in m$^3$/s; $q_u$ is the unit peak discharge in m$^3$/s per cm of runoff per km$^2$ of area; $A$ is in km$^2$; $P_e$ is the 24-hour rainfall excess in cm for a given return period; and $F_p$ is a dimensionless adjustment factor (see Table 5-7) to account for ponds and swamps that are not in the primary flow path. The depth of runoff, $P_e$, is determined by the curve number method and can be expressed as

$$P_e = \frac{(P_{24} - 0.2S_r)^2}{P_{24} + 0.8S_r} \quad \text{for } P_{24} > 0.2S_r$$  \hspace{1cm} (5-33)

where $P_{24}$ is the 24-hour rainfall, and $S_r$ is the potential maximum retention of the soil. The parameter $q_u$ can be obtained from peak discharge curves provided by the NRCS (SCS, 1986) or can be approximated by the empirical equation

$$\log q_u = C_0 + C_1 \log t_c + C_2 \left(\log t_c\right)^2 - 2.366$$  \hspace{1cm} (5-34)

when $t_c$ ranges from 0.1 to 10 hours. In Equation 5-34, $t_c$ is in hours and is computed using one of the NRCS models described earlier. Regression constants $C_0$, $C_1$, and $C_2$ can be obtained in Table 5-8, in which $I_a$ is the initial abstraction, evaluated as

$$I_a = 0.2S_r$$  \hspace{1cm} (5-35)

When $I_a/P_{24} < 0.1$, values of $C_0$, $C_1$, and $C_2$ corresponding to $I_a/P_{24} = 0.1$ should be used, and if $I_a/P_{24} > 0.5$, values of $C_0$, $C_1$, and $C_2$ corresponding to $I_a/P_{24} = 0.5$ should be used. The rainfall type (i.e., I, IA, II, and III) refers to geographic regions shown in Figure 5-2.
Table 5-7: Pond and swamp adjustment factor

<table>
<thead>
<tr>
<th>Percentage of pond and swamp areas</th>
<th>$F_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.00</td>
</tr>
<tr>
<td>0.2</td>
<td>0.97</td>
</tr>
<tr>
<td>1.0</td>
<td>0.87</td>
</tr>
<tr>
<td>3.0</td>
<td>0.75</td>
</tr>
<tr>
<td>5.0</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Note: If $> 5.0$, consider routing runoff through ponds and swamps

Table 5-8: Constants for computing unit peak discharge

<table>
<thead>
<tr>
<th>Rainfall Type</th>
<th>$I_o/P_{2d}$</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.10</td>
<td>2.30550</td>
<td>-0.51429</td>
<td>-0.11750</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>2.23527</td>
<td>-0.50387</td>
<td>-0.08929</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>2.18219</td>
<td>-0.48488</td>
<td>-0.06589</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>2.10624</td>
<td>-0.45695</td>
<td>-0.02835</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>2.00303</td>
<td>-0.40769</td>
<td>0.01983</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>1.87733</td>
<td>-0.32274</td>
<td>0.05754</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>1.76312</td>
<td>-0.15644</td>
<td>0.00453</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>1.67889</td>
<td>-0.06930</td>
<td>0.0</td>
</tr>
<tr>
<td>IA</td>
<td>0.10</td>
<td>2.03250</td>
<td>-0.31583</td>
<td>-0.13748</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>1.91978</td>
<td>-0.28215</td>
<td>-0.07020</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>1.83842</td>
<td>-0.25543</td>
<td>-0.02597</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>1.72657</td>
<td>-0.19826</td>
<td>0.02633</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>1.63417</td>
<td>-0.09100</td>
<td>0.0</td>
</tr>
<tr>
<td>II</td>
<td>0.10</td>
<td>2.55323</td>
<td>-0.61512</td>
<td>-0.16403</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>2.46532</td>
<td>-0.62257</td>
<td>-0.11657</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>2.41896</td>
<td>-0.61594</td>
<td>-0.08820</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>2.36409</td>
<td>-0.59857</td>
<td>-0.05621</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>2.29238</td>
<td>-0.57005</td>
<td>-0.02281</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>2.20282</td>
<td>-0.51599</td>
<td>-0.01259</td>
</tr>
<tr>
<td>III</td>
<td>0.10</td>
<td>2.47317</td>
<td>-0.51848</td>
<td>-0.17083</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>2.39628</td>
<td>-0.51202</td>
<td>-0.13245</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>2.35477</td>
<td>-0.49735</td>
<td>-0.11985</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>2.30726</td>
<td>-0.46541</td>
<td>-0.11094</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>2.24876</td>
<td>-0.41314</td>
<td>-0.11508</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>2.17772</td>
<td>-0.36803</td>
<td>-0.09525</td>
</tr>
</tbody>
</table>
5.3.3 USGS Regression Models

On behalf of the United States Geological Survey (USGS), Sauer et al. (1981) evaluated data from nearly 270 urban watersheds to develop a series of regression equations for peak discharge. The equations are applicable to basins characterized as at least 15 percent residential, commercial, or industrial land cover. For various return periods, they can be expressed as

\[ UQ_2 = 2.35 A^{0.41} S_L^{0.17} (P_2 + 3)^{2.04} (S_T + 8)^{-0.65} (13 - DF)^{-0.32} P^{0.15} RQ_2^{0.47} \] (5-36)

\[ UQ_5 = 2.70 A^{0.35} S_L^{0.16} (P_2 + 3)^{1.86} (S_T + 8)^{-0.59} (13 - DF)^{-0.31} P^{0.11} RQ_5^{0.54} \] (5-37)

\[ UQ_{10} = 2.95 A^{0.32} S_L^{0.15} (P_2 + 3)^{1.75} (S_T + 8)^{-0.57} (13 - DF)^{-0.30} P^{0.09} RQ_{10}^{0.58} \] (5-38)

\[ UQ_{25} = 2.78 A^{0.31} S_L^{0.15} (P_2 + 3)^{1.76} (S_T + 8)^{-0.54} (13 - DF)^{-0.29} P^{0.07} RQ_{25}^{0.60} \] (5-39)
where $UQ_i$ and $RQ_i$ are the urban and rural peak discharges for return period $i$, respectively, in cfs; $A$ is in $\text{mi}^2$; $S_L$ is the channel slope in ft/mi between points that are 10 percent and 85 percent of the distance along the channel from the basin outlet to the watershed boundary; $P_2$ is the rainfall in inches for a 2-year, 2-hour storm; $S_T$ is the percent of basin storage found in lakes and reservoirs; $DF$ is a basin development factor that is indicative of drainage efficiency; and $p$ is the percentage of impervious cover.

In applying the USGS regression equations, $RQ_i$ can be obtained from USGS reports for the study area and $DF$ can be obtained by qualitatively evaluating drainage characteristics of the basin. With respect to the latter, the basin is divided into three sub-basins, and the presence of the following four conditions should be considered for each sub-basin: (1) channel improvements (e.g., straightening or deepening); (2) impervious channel linings; (3) secondary tributaries consisting of storm drains or sewers; and (4) urbanized conditions in which streets and highways include curbs and gutters. For each of these, a zero or one is assigned based on whether the condition is less than or greater than 50 percent prevalent, and the values are summed. For example, consider three identical sub-basins, within which 60 percent of the main drainage channel and tributaries have been improved and are lined with impervious material, 20 percent of secondary tributaries consist of storm sewers, and 15 percent of the basin has been urbanized, with the majority of streets having curbs and gutters. For each sub-basin, the first and second conditions would be assigned values of unity, while conditions (3) and (4) would be assigned values of zero. The corresponding value of $DF$ is then $3 \times (1 + 1 + 0 + 0)$, or 6. Note that the maximum and minimum bounds of $DF$ are 12 and zero, respectively. A value of zero, however, does not indicate that no effects of urbanization exist; it merely indicates that the effects are not prevalent.

The watersheds used in development of the USGS regression equations were all less than 100 $\text{mi}^2$ (260 $\text{km}^2$) and had slopes ranging from 3.0 to 70 ft/mi (0.6 m/km to 13 m/km). In addition, rainfall intensities evaluated were less than 3.0 inches (75 mm). These values should be considered as estimated limits of application.
5.4 CONTINUOUS-FLOW MODELS

For larger watersheds in which channel storage may be significant, simple peak-flow methods are not sufficient for the evaluation of runoff. In these cases, response to rainfall events tends to be slower, and it is necessary to evaluate the variation of runoff over time (i.e., the entire hydrograph). Hydrographs are also necessary for watersheds in which significant variability exists in land use (e.g., urbanization), soil types, or topography. In this respect, unit hydrograph models are the most widely-used techniques for evaluating runoff.

5.4.1 Unit Hydrograph Approach

A unit hydrograph is a linear conceptual model that can be used to transform rainfall excess into a runoff hydrograph. By definition, it is the hydrograph that results from 1 inch, or 1 cm in S.I. units, of rainfall excess generated uniformly over the watershed at a uniform rate during a specified period of time. The process by which an existing unit hydrograph is used with given storm inputs to yield a direct runoff hydrograph is known as convolution. In discretized form, the convolution equation can be expressed as (Chow et al., 1988)

\[ Q_n = \sum_{m=1}^{n-1} P_m U_{n-m+1} \quad \text{for } n = 1, 2, ..., N \]  

(5-43)

where \( Q_n \) is a direct runoff hydrograph ordinate; \( P_m \) is excess rainfall at interval \( m \); and \( U_{n-m+1} \) represents the unit hydrograph ordinate. Subscripts \( n \) and \( m \) refer to the runoff hydrograph time interval and precipitation time interval, respectively. Note that if total rainfall data have been recorded, initial abstractions, such as interception storage, depression storage, and infiltration should be subtracted to define only the excess rainfall distribution. The curve number method (see Chapter 3) is a popular technique for estimating rainfall excess.

To develop a unit hydrograph from measured data, a gaged watershed ranging in size from 1.0 and 1,000 mi\(^2\) (2.6 and 2,600 km\(^2\)) should be selected. Assuming that a sufficient number of rainfall-runoff records can be obtained for the watershed, selection of specific events to use in the analysis should be made in accordance with the following criteria (Viessman and Lewis, 1996):

- Storms should have a simple structure (i.e., individually occurring) with relatively uniform spatial and temporal rainfall distributions;
- Direct runoff should range from 0.5 to 1.75 inches (1.25 to 4.5 cm);
• Duration of the rainfall event should range from 10 to 30 percent of the lag time, defined as the time from the midpoint of the excess rainfall to the peak discharge; and

• At least several storms that meet the previous criteria and that have a similar duration of excess rainfall should be analyzed to obtain average rainfall-runoff data.

Once data are selected, the measured time distribution of rainfall excess, $P$, and direct runoff, $Q$, are applied within a reverse convolution, or deconvolution, process to derive the unit hydrograph. Assuming there are $M$ discrete values of excess rainfall that define a storm event and $N$ discrete values of direct runoff, then from Equation 5-43, $N$ equations can be written for $Q_n$, $n = 1, 2, \ldots, N$, in terms of $N - M + 1$ unit hydrograph ordinates (Mays, 2001). For example,

\[
\begin{align*}
Q_1 &= P_1 U_1 \\
Q_2 &= P_2 U_1 + P_1 U_2 \\
& \vdots \\
Q_M &= P_M U_1 + P_{M-1} U_2 + \ldots + P_1 U_M \\
Q_{M+1} &= 0 + P_M U_2 + \ldots + P_2 U_M + P_1 U_{M+1} \\
& \vdots \\
Q_{N-1} &= 0 + 0 + \ldots + 0 + 0 + \ldots + P_M U_{N-M} + P_{M-1} U_{N-M+1} \\
Q_N &= 0 + 0 + \ldots + 0 + 0 + \ldots + 0 + P_M U_{N-M+1}
\end{align*}
\]

(5-44)

represents a set of $N$ equations with $N - M + 1$ unknowns that can be solved algebraically or by matrix operators. As a final step, since averaged data was used in the analysis, the unit hydrograph should be adjusted to ensure that the distribution corresponds to 1 inch, or 1 cm, of direct runoff.

### 5.4.1.1 S-Hydrograph Method

Depending on the unit hydrograph application, the current duration of excess rainfall may not be convenient. For example, it is necessary to divide the design storm into a discrete number of time intervals. The duration that results from basin parameters will often not evenly divide into the design storm duration. In other cases, the effective duration may be of such magnitude that the number of computations can be significantly reduced if a larger duration is used. Fortunately, the S-hydrograph method can be used to convert a unit hydrograph of any given duration into a unit hydrograph of any other effective duration. The S-hydrograph, or S-curve, theoretically represents the response of a
particular watershed to a constant rainfall excess for an indefinite period. It can be derived by adding an infinite series of lagged unit hydrographs, as shown in Figure 5-3. In application, the S-hydrograph computed with the $t_d$ duration unit hydrograph is first lagged by the desired duration, $t_d'$. The difference between the two S-curves is then multiplied by the ratio $t_d' / t_d$. The result is a new unit hydrograph having an effective duration of $t_d'$.

![Figure 5-3: S-hydrograph method](image)

### 5.4.2 Synthetic Unit Hydrographs

In many applications, the measured data required to develop a unit hydrograph is unavailable or does not exist. In these cases, synthetic unit hydrograph procedures can be used to develop unit hydrographs for ungaged locations in a watershed or for other watersheds that have similar characteristics.

#### 5.4.2.1 NRCS Dimensionless Unit Hydrograph

The NRCS dimensionless unit hydrograph, tabulated in Table 5-9 and illustrated in Figure 5-4, was developed based on data from a large number of watersheds (SCS, 1985). The dimensionless time and runoff ordinates can be dimensionalized by multiplying the corresponding values (i.e., $t/t_p$ or $Q/Q_p$) by time from the beginning of excess rainfall to the time of peak discharge, $t_p$, or the peak runoff, respectively.
Table 5-9: NRCS dimensionless unit hydrograph

<table>
<thead>
<tr>
<th>$t/t_p$</th>
<th>$Q/Q_p$</th>
<th>$t/t_p$</th>
<th>$Q/Q_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.000</td>
<td>1.4</td>
<td>0.780</td>
</tr>
<tr>
<td>0.1</td>
<td>0.030</td>
<td>1.5</td>
<td>0.680</td>
</tr>
<tr>
<td>0.2</td>
<td>0.100</td>
<td>1.6</td>
<td>0.560</td>
</tr>
<tr>
<td>0.3</td>
<td>0.190</td>
<td>1.8</td>
<td>0.390</td>
</tr>
<tr>
<td>0.4</td>
<td>0.310</td>
<td>2.0</td>
<td>0.280</td>
</tr>
<tr>
<td>0.5</td>
<td>0.470</td>
<td>2.2</td>
<td>0.207</td>
</tr>
<tr>
<td>0.6</td>
<td>0.660</td>
<td>2.4</td>
<td>0.147</td>
</tr>
<tr>
<td>0.7</td>
<td>0.820</td>
<td>2.6</td>
<td>0.107</td>
</tr>
<tr>
<td>0.8</td>
<td>0.930</td>
<td>2.8</td>
<td>0.077</td>
</tr>
<tr>
<td>0.9</td>
<td>0.990</td>
<td>3.0</td>
<td>0.055</td>
</tr>
<tr>
<td>1.0</td>
<td>1.000</td>
<td>3.5</td>
<td>0.025</td>
</tr>
<tr>
<td>1.1</td>
<td>0.990</td>
<td>4.0</td>
<td>0.011</td>
</tr>
<tr>
<td>1.2</td>
<td>0.930</td>
<td>4.5</td>
<td>0.005</td>
</tr>
<tr>
<td>1.3</td>
<td>0.860</td>
<td>5.0</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Source: SCS (1985)

Figure 5-4: NRCS dimensionless unit hydrograph (SCS, 1985)
Based on NRCS recommendations, time-to-peak discharge is a function of and highly sensitive to time of concentration. The relationship between these variables can be expressed as

\[ t_p = \frac{2}{3} t_c \]  \hspace{1cm} (5-45)

where \( t_c \) should be computed using one of the NRCS formulas discussed previously and \( Q_p \), in cfs/in or m³/s/m is defined as

\[ Q_p = \frac{K_p A}{t_p} \]  \hspace{1cm} (5-46)

where \( A \) is in mi² or km²; \( K_p \) is a constant equal to 484 in U.S. customary units and 2.08 in S.I. units; and \( t_p \) is given in hours. The time associated with the recession limb of the unit hydrograph, or time from peak discharge to the end of direct runoff, can be approximated multiplying \( t_p \) by 4.0. Note that the resulting synthetic unit hydrograph is applicable only for an effective duration of excess rainfall, \( t_d \), given as (SCS, 1985)

\[ t_d = 0.133 t_c \]  \hspace{1cm} (5-47)

In addition, the NRCS dimensionless hydrograph assumes that 3/8 of the total runoff volume occurs before the time at which the peak discharge occurs. In mountainous regions, more than 3/8 of runoff volume may occur under the rising limb, causing the constant in Equation 5-46 (i.e., \( K_p \)) may increase to values near 600. For flat, marshy areas, the constant may be less than 300 (McCuen, 1998).

### 5.4.2.2 NRCS Triangular Unit Hydrograph

The NRCS triangular unit hydrograph, as shown in Figure 5-5, is an approximation to the NRCS dimensionless unit hydrograph. The time-to-peak discharge, peak flow, and effective rainfall duration are determined using Equations 5-45 through 5-47. The only difference from the curvilinear method is that the time from peak discharge to the end of direct runoff is equal to \( 1.67 t_p \). The triangular hydrograph is attractive because of its simplicity: only three parameters are needed to define the entire unit hydrograph.
5.4.2.3 Snyder Unit Hydrograph

Based on a study of watersheds located in the Appalachian mountains, the method proposed by Snyder (1938) for unit hydrograph synthesis is derived from relationships between the characteristics of a standard unit hydrograph and descriptors of basin morphology. It relates the time from the centroid of the excess rainfall to the peak of the unit hydrograph (i.e., lag time) to geometric characteristics of the basin in order to derive critical points for interpolating the unit hydrograph. Lag time is evaluated by

\[ t_L = C_1 C_t (L L_{Ct})^{0.3} \]  

(5-48)

where \( t_L \) is in hours; \( C_t \) is a constant equal to 1.0 in U.S. customary units and 0.75 in S.I. units; \( C_t \) is an empirical watershed storage coefficient, which generally ranges from 1.8 to 2.2; \( L \) is the length of the main stream channel in mi or km; and \( L_{Ct} \) is the length of stream channel from a point nearest the center of the basin to the outlet, or other point of interest, in mi or km.

The standard duration of excess rainfall is computed empirically by

\[ t_d = \frac{t_L}{5.5} \]  

(5-49)

Adjusted values of lag time, \( t_{La} \), for other durations of rainfall excess can be obtained by
where $t_{da}$ is the alternative unit hydrograph duration. Time-to-peak discharge can be computed as a function of lag time and duration of excess rainfall, expressed as

$$t_p = t_{La} + 0.5t_{da}$$  \hfill (5-51)

The corresponding peak discharge, $Q_p$ is

$$Q_p = \frac{C_2C_pA}{t_{La}}$$  \hfill (5-52)

where $Q_p$ is in cfs/in or m$^3$/s/m; $C_2$ is a constant equal to 640 in U.S. customary units and 2.75 in S.I. units; $A$ is in m$^2$ or km$^2$; $t_{La}$ is in hours; and $C_p$ is an empirical constant ranging from 0.5 to 0.7. Note that coefficients $C_t$ and $C_p$ are regional parameters that should be calibrated or be based on values obtained for similar gaged drainage areas. The ultimate shape of Snyder’s unit hydrograph is primarily controlled by two parameters, $W_{50}$ and $W_{75}$, representing widths of the unit hydrograph at discharges equal to 50 and 75 percent of the peak discharge, respectively. These shape parameters can be evaluated by

$$W_{50} = C_{50}\left(\frac{A}{Q_p}\right)^{1.08}$$  \hfill (5-53)

and

$$W_{75} = C_{75}\left(\frac{A}{Q_p}\right)^{1.08}$$  \hfill (5-54)

where $C_{50}$ is a constant equal to 770 in U.S. customary units and 2.14 in S.I. units, and $C_{75}$ is a constant equal to 440 in U.S. customary units and 1.22 in S.I. units. The location of the end points for $W_{50}$ and $W_{75}$ are often placed such that one-third of both values occur prior to the time-to-peak discharge and the remaining two-thirds occur after the time to peak. Finally, the base time, $t_b$, or
time from beginning to end of direct runoff, should be evaluated such that the unit hydrograph represents 1 inch (or 1 cm in S.I. units) of direct runoff volume. However, a rough estimate for $t_b$ for small watersheds is three to five times the time-to-peak discharge. With known values of $t_p$, $Q_p$, $W_{50}$, and $W_{75}$, along with the adjusted base time, one can then locate a total of seven unit hydrograph ordinates, as shown in Figure 5-6.

![Snyder unit hydrograph](image)

**Figure 5-6: Snyder unit hydrograph**

### 5.4.2.4 Clark Unit Hydrograph

Time-area methods attempt to relate travel time to a portion of a watershed that may contribute runoff during that time. One of the most common time-area techniques is that derived by Clark (1945). This method explicitly represents the processes of translation and attenuation, the two key factors in transformation of excess rainfall to a runoff hydrograph. Translation refers to the movement by gravity, without storage effects; whereas attenuation refers to the reduction of runoff magnitude due to resistance forces and the storage effects of the soil, channel, and land surfaces. The method is based on the concept that translation of flow through the watershed can be described by runoff isochrones and the corresponding histogram of contributing area versus time, as shown in Figure 5-7. Isochrones are lines of equal travel time and describe the fraction of watershed area contributing runoff to the watershed outlet as a function of travel time. For example, Figure 5-7 shows that the number of sub-areas contributing runoff varies throughout a rainfall event, with
full contribution from all areas for \( t \geq t_6 \). Note that each sub-area is delineated so that all precipitation falling on it has the same time of travel to the outflow point.

Developing a time-area curve for a watershed can be difficult in practice. For those watersheds that lack a time-area relationship, HEC-HMS developed by the U.S. Army Corps of Engineers uses the following relationship (HEC, 2000):

\[
\frac{A_{c,t}}{A_T} = \begin{cases} 
  1.414 \left( \frac{t}{t_c} \right)^{1.5} & \text{for } t \leq \frac{t_c}{2} \\
  1 - 1.414 \left( 1 - \frac{t}{t_c} \right)^{1.5} & \text{for } t > \frac{t_c}{2}
\end{cases}
\] (5-55)

where \( A_{c,t} \) is the cumulative watershed area contributing at time \( t \), and \( A_T \) is the total watershed area. If the sub-areas (i.e., \( A_i \)) are multiplied by a unit depth of excess rainfall and divided by \( \Delta t \), the computational time step, the result is a translated hydrograph that represents inflow to a conceptual linear reservoir located at the watershed outlet.

![Time-area isochrones and histogram for Clark’s method](image)

Figure 5-7: Time-area isochrones and histogram for Clark’s method

To account for attenuation, the translated hydrograph is routed through the conceptual linear reservoir, assumed to possess storage properties similar to
those of the watershed. The routing model is based on a mass balance equation, expressed as

$$\frac{dS_t}{dt} = I_t - Q_t$$  \hspace{1cm} (5-56)

where $dS_t/dt$ is time rate of change of water in storage, and $I_t$ and $Q_t$ are average inflow and outflow, respectively, at time $t$. For a linear reservoir model, storage at a particular time is related to outflow by

$$S_t = K_s Q_t$$  \hspace{1cm} (5-57)

where $K_s$ is a constant that represents storage effects of the watershed and is often approximated by the watershed lag time, $t_L$. Combining and solving Equations 5-56 and 5-57 using a simple finite difference approximation leads to

$$Q_t = C_1 I_t + C_2 Q_{t-1}$$  \hspace{1cm} (5-58)

where $C_1$ and $C_2$ are routing coefficients evaluated by

$$C_1 = \frac{\Delta t}{K_s + 0.5 \Delta t}$$  \hspace{1cm} (5-59)

and

$$C_2 = 1 - C_1$$  \hspace{1cm} (5-60)

The average outflow during period $\Delta t$ is

$$\overline{Q_t} = \frac{Q_{t-1} + Q_t}{2}$$  \hspace{1cm} (5-61)

If the inflow ordinates are runoff from a unit depth of excess rainfall, the average outflows derived by Equation 5-61 represent Clark’s unit hydrograph ordinates. Clark’s unit hydrograph is, therefore, obtained by routing a unit depth of direct runoff to a channel in proportion to the time-area curve and
routing the resulting discharge through a linear reservoir. Note that solution of
Equations 5-58 and 5-61 is a recursive process. As such, average outflow
ordinates of the unit hydrograph will theoretically continue for an infinite
duration. It is, therefore, customary to truncate the recession limb of the unit
hydrograph at a point where outflow volume exceeds 0.995 inches or cm.
Clark’s method is based on the premise that the duration of the rainfall excess is
infinitesimally small. Because of this, Clark’s unit hydrograph is referred to as
an instantaneous unit hydrograph (IUH). The IUH is highly indicative of
watershed storage characteristics since rainfall-duration effects are essentially
eliminated. In practical applications, it is usually necessary to transform the
IUH into a unit hydrograph of specific duration. This can be accomplished by
lagging the IUH by the desired duration and averaging the ordinates.

5.4.2.5 Tri-Triangular Unit Hydrograph

The tri-triangular method, depicted in Figure 5-8, is frequently used to derive
rainfall dependent inflow/infiltration (RDII) flows for urban drainage systems.
The technique involves summing the corresponding ordinates of up to three
triangular hydrographs to derive a single unit hydrograph. Each of the three
triangular hydrographs has its own characteristic parameters: time-to-peak flow,
\( t_i \), a recession constant, \( K_i \), and fraction of the total excess rainfall volume
allocated to the triangle, \( \forall_i \).

The three triangular hydrographs are conceptual representations of different
components of direct runoff or RDII. The first triangle represents rapidly
responding components such as contributions from pavements and rooftops, or
direct inflow or rapid infiltration into separate sewer systems. The third triangle
represents slow runoff components such as ground water contributions or slow
infiltration into sewers. The second triangle represents runoff or infiltration
with a medium time response. The time-to-peak flow for the first hydrograph
typically varies between 1 and 2 hours, depending on the size of the tributary
area, and \( t \) values for the second hydrograph ranges from 4 to 8 hours. For the
third, the parameter varies greatly depending on the infiltration characteristics
of the system being modeled, but generally range from 10 to 24 hours.
Recession constants typically range from 2 to 3 for the first triangle and from 2
to 4 for the second and third hydrographs.

5.4.2.6 Espey-Altman Unit Hydrograph

Espey and Altman (1978) evaluated 10-minute unit hydrographs resulting from
a series of rainfall excesses over 41 watersheds ranging in size from 9 acres to
15 mi\(^2\) (4 ha to 39 km\(^2\)). The unit hydrographs were used to derive the
following empirical relationships:
Figure 5-8: The tri-triangular unit hydrograph

\[ t_p = 3.1 \frac{L^{0.23} \phi^{1.57}}{S^{0.25} P^{0.18}} \]  \hspace{1cm} (5-62)

\[ Q_p = 31.62 \times 10^3 \frac{A^{0.96}}{t_p^{1.07}} \]  \hspace{1cm} (5-63)

\[ t_b = 125.89 \times 10^3 \frac{A}{Q_p^{0.93}} \]  \hspace{1cm} (5-64)

\[ W_{50} = 16.22 \times 10^3 \frac{A^{0.93}}{Q_p^{0.92}} \]  \hspace{1cm} (5-65)
$W_{75} = 3.24 \times 10^3 \frac{A^{0.79}}{Q_p^{0.78}}$  

(5-66)

where $L$ is in ft; $\phi$ is a dimensionless conveyance factor that ranges from 0.8 for concrete lined channels to 1.3 for natural channels (Bedient and Huber, 2002); $S$ is channel slope in ft/ft for the lower 80 percent of the flow path; $p$ is percent imperviousness; $A$ is in mi$^2$; and other terms are as previously defined, with $t_p$, $t_b$, $W_{50}$, and $W_{75}$ in minutes and $Q_p$ is cfs/in. The location of the end points for $W_{50}$ and $W_{75}$ are assigned so that one-third of both values occur prior to $t_p$ and the remaining two-thirds occur after $t_p$. With characteristics of the watershed known, Equations 5-62 through 5-66 can be used to locate seven points that comprise the Espey-Altman synthetic unit hydrograph, similar to that shown in Figure 5-6.

### 5.4.2.7 Colorado Urban Hydrograph

The Colorado Urban Hydrograph Procedure (CUHP) is an adaptation of Snyder’s unit hydrograph based on data for Colorado urban watersheds ranging in size from 100 to 200 acres (40 to 80 ha) (UDFCD, 1984). The technique is commonly used in the state of Colorado to derive unit hydrographs for urban and rural watersheds that have areas ranging from 90 acres to 5 mi$^2$ (36 ha to 13 km$^2$). Whenever a larger watershed is studied, it is recommended that the basin be subdivided into subcatchments of 5 mi$^2$ (13 km$^2$) or less. The shape of the CUHP unit hydrograph is determined based on physical characteristics of the watershed. The method incorporates the effects of watershed size, shape, percentage of imperviousness, length of the primary drainage channel, slope, and other factors.

To apply the method, lag time of a watershed is determined by

$$t_L = C_t \left( \frac{L L_{CA}}{\sqrt{S}} \right)^{0.48}$$  

(5-67)

where $t_L$ is in hours; $L$ is length in miles along the drainageway from the outlet or other point of interest to the upstream watershed boundary; $L_{CA}$ is channel length in miles from a point nearest the center of the basin to the outlet; $S$ is the length weighted average slope of the catchment along the drainage path; and $C_t$ is an empirical coefficient. Once the lag time is determined, the time-to-peak flow can be obtained by adding 0.5$t_d$ to the lag time in consistent units. Peak flow rate is computed as
\[ Q_p = \frac{640C_pA}{I_p} \]  

(5-68)

where \( Q_p \) is in cfs; \( A \) is in \( \text{mi}^2 \); \( C_p \) is a unit hydrograph peaking coefficient and is determined by

\[ C_p = P_f C_t A^{0.15} \]  

(5-69)

where \( P_f \) is peaking parameter. \( C_t \) and \( P_f \) are defined in terms of percent impervious, \( p \), of the catchment as

\[ C_t = a p^2 + b p + c \]  

(5-70)

and

\[ P_f = d p^2 + e p + f \]  

(5-71)

The regression coefficients \( a, b, c, d, e, \) and \( f \) are defined in terms of \( p \) in Table 5-10. The capability of the CUHP to account for percent imperviousness can be viewed as a certain improvement over Snyder’s method.

| Table 5-10: CUHP coefficients as a function of percent imperviousness |
|------------------------|--------|--------|--------|--------|--------|--------|
| \( p \)               | \( a \) | \( b \) | \( c \) | \( D \) | \( e \) | \( f \) |
| \( p \leq 10 \)        | 0.0    | -0.00371 | 0.163  | 0.00245 | -0.012 | 2.16   |
| \( 10 \leq p \leq 40 \) | \( 2.3 \times 10^{-5} \) | -0.00224 | 0.146  | 0.00245 | -0.012 | 2.16   |
| \( p \geq 40 \)        | \( 3.3 \times 10^{-5} \) | -0.000801 | 0.120  | -0.00091 | 0.228 | -2.06  |

The widths of the unit hydrograph at 50 and 75 percent of the peak are estimated as

\[ W_{50} = \frac{500}{Q_p} \left( \frac{A}{Q_p} \right) \]  

(5-72)

and
where \( W_{50} \) and \( W_{75} \), defined in Figure 5-6, are in hours; \( Q_p \) is in cfs; and \( A \) is in \( \text{mi}^2 \). As a general rule, the smaller of 0.35\( W_{50} \) and 0.6\( t_p \) is assigned to the left of \( t_p \) at 50 percent of the peak flow, and 0.65\( W_{50} \) is assigned to the right of the peak flow. The width assigned to the left \( t_p \) at 75 percent of the peak flow depends on the case used for allocation of \( W_{50} \). If 0.35\( W_{50} \) was assigned to the left at 50 percent of the peak flow, then 0.45\( W_{75} \) is allocated to the left side at 75 percent of the peak flow. Otherwise, the left width at 75 percent of the peak flow will be 0.424\( t_p \). The right side of the peak, however, is always equal to 0.55\( W_{75} \).

### 5.4.2.8 Delmarva Unit Hydrograph

The Delmarva unit hydrograph (Welle et al., 1980) is an adaptation of the NRCS dimensionless unit hydrograph to better represent the runoff characteristics of the Delmarva Peninsula, which consists of the states of Delaware, Maryland, and Virginia. The flat topography of the Delmarva Peninsula and availability of considerable surface storage causes the shape of observed storm hydrographs to be significantly different from those generated using the NRCS unit hydrographs. The Delmarva unit hydrograph produces lower peak flow rate than the NRCS unit hydrographs, but yields the same flow volume. The dimensionless time (i.e., \( t/t_p \)) and runoff ordinates (i.e., \( Q/Q_p \)) for the Delmarva unit hydrograph are given in Table 5-11 and Figure 5-9.

The Delmarva unit hydrograph uses Equation 5-46 to estimate peak flow rate. When compared with peak flow estimation for the NRCS unit hydrographs, however, the constant \( K_p \) is set to 284 (U.S. customary units) for the Delmarva unit hydrograph. Thus, peak flow is expressed as

\[
Q_p = \frac{284 A}{t_p} \tag{5-74}
\]

where \( Q_p \) is peak flow rate in cfs; \( A \) is area of the watershed in \( \text{mi}^2 \); and \( t_p \) is time to peak of the unit hydrograph in hours calculated according to Equation 5-45. The time base of the Delmarva unit hydrograph is approximately 10\( t_p \).
### Table 5-11: Delmarva unit hydrograph

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Source: Welle et al. (1980)
5.4.3 Santa Barbara Urban Hydrograph

The Santa Barbara Urban Hydrograph (SBUH) model was developed for the Santa Barbara County Flood Control and Water Conservation District (Stubchaer, 1980). The method combines runoff from impervious and pervious regions to develop a direct runoff hydrograph, which is subsequently routed through a conceptual reservoir that delays outflow by a period equal to the time of concentration. Application begins by computing the instantaneous runoff (i.e., reservoir inflow), $I$, expressed as

$$I = A\left[ip_i + i_e(1.0 - p_i)\right] \tag{5-75}$$

where $A$ is the basin area; $i$ is rainfall intensity; $p_i$ is the fraction of imperviousness; and $i_e$ is the average excess rainfall rate over $\Delta t$. The runoff hydrograph is then obtained by

$$Q_t = Q_{t-1} + \left(\frac{\Delta t}{2t_e + \Delta t}\right)(I_{t-1} + I_t - 2Q_{t-1}) \tag{5-76}$$

where $Q_t$ and $Q_{t-1}$ are ordinates of the resulting runoff hydrograph at times $t$ and $t-1$, respectively, and $I_t$ and $I_{t-1}$ are instantaneous runoff rates at $t$ and $t-1$, respectively. The primary limitation of the SBUH model is that the calculated
peak flow cannot occur after rainfall ceases. This may at times be problematic, particularly for short-duration storms over large, flat watersheds.

5.4.4 San Diego Modified Rational Hydrograph

As previously described, the rational method is a widely used technique for estimation of peak flow. In addition to peak flows, water resources applications such as the design of detention basins require knowledge of the runoff hydrograph to determine total inflow volume. Adopted by the San Diego County, the San Diego modified rational hydrograph derives a direct runoff hydrograph using the peak flow estimated using rational formula (see Equation 5-29) for watersheds no larger than 1 mi² in size.

The San Diego modified rational hydrograph is developed to work with a 6-hour storm event. The method starts by creating a rainfall distribution of the 6-hour storm whereby each rain block lasts for a duration equal to the time of concentration. To distribute the storm event, first, the center of the peak rainfall block is placed at the 4-hour position. Then, the other blocks are placed using the so-called (2/3, 1/3) approach in which rainfall blocks are distributed in a sequence, alternating two blocks to the left and one block to the right of the 4-hour time. Rainfall depth for each rainfall block is calculated as

\[ P_N = 0.124 P_{6-hr} \left( \frac{N \cdot t_c}{0.355} \right) \left( N - 1 \right)^{0.355} \]  (5-77)

where \( P_N \) is rainfall depth for each rainfall block in inch or mm; \( P_{6-hr} \) is six-hour storm depth in inch or mm; \( t_c \) is time of concentration in minutes; and \( N \) is an integer representing position of a given rainfall block. \( N \) is 1 for the peak rainfall block, and is assigned according to the (2/3, 1/3) distribution rule for other rainfall blocks.

Once the rainfall distribution is established, a triangular hydrograph is developed for each block of rain. Peak flow for the triangular hydrograph is determined using the rational formula, and the time base of the hydrograph will be \( 2t_c \). Finally, the runoff hydrograph for the 6-hr storm event is determined by adding all the triangular hydrographs from each block of rain. The final hydrograph will have its peak at 4 hours plus half of the \( t_c \) and total volume under the hydrograph will be the product of runoff coefficient, the six-hour precipitation depth and the subcatchment area.

5.4.5 Nonlinear Reservoir Model

The nonlinear reservoir model simulates a watershed as a shallow reservoir in which inflow is equal to the rainfall excess and outflow, \( Q \), is a nonlinear
function of the depth of flow. The difference between inflows and outflow then represents the rate of storage (i.e., Equation 5-56). The method proposed by Huber and Dickinson (1988), which is based on the Manning equation, describes outflow as

\[ Q = \frac{K_m B}{n} (y - y_d)^{5/3} S^{1/2} \]  

(5-78)

where \( K_m \) is a constant equal to 1.49 in U.S. customary units and 1.0 in S.I. units; \( B \) is the representative width of the basin; \( n \) is the Manning roughness coefficient, which can be read from Table 5-1; \( y \) and \( y_d \) are water depth and depression storage, respectively; and \( S \) is watershed slope in ft/ft or m/m. Incorporating Equation 5-78 into a mass balance on inflows and outflows yields

\[ \frac{y_2 - y_1}{\Delta t} = i_e - \frac{K_m B S^{1/2}}{A n} \left( \frac{y_1 + y_2}{2} - y_d \right)^{5/3} \]  

(5-79)

\( y_1 \) and \( y_2 \) are depths at the beginning and end of the time step, \( \Delta t \), and \( i_e \) is the excess rainfall intensity during the same period. Note that this expression requires an iterative solution for \( y_2 \), which can then be used with Equation 5-78 to evaluate \( Q \).

5.4.6 Kinematic-Wave Model

As discussed earlier in Section 5.2.1.1, the kinematic wave equation (i.e., Equation 5-7) is based on a combination of the full continuity equation and the momentum equation for steady, uniform flow. As a result, it is limited to cases in which flow attenuation is not significant. For such circumstances, unit width flow at a basin outlet can be computed by substituting Equation 5-10 into 5-2, which yields

\[ q = \alpha_k (i t)^n \]  

(5-80)

where \( i \) is, for the time being, assumed to be constant. This relationship, however, applies only to the rising limb of the hydrograph. The full temporal distribution of runoff requires an additional comparison between the duration of rainfall excess, \( t_d \), and the time of concentration, \( t_c \). The reasoning here is that when rainfall ceases, intensity approaches zero and any accumulation on upstream portions of the basin travel to the outlet at a rate equal to the
kinematic wave celerity, $c_k$, defined in Equation 5-8. Integration of that equation beyond $t_d$ yields an implicit relationship for $q$, expressed as

$$L - \frac{q}{i} = -m\alpha_k^{(m)}q^{-1/m}(t - t_d) = 0 \quad (5-81)$$

Three cases are possible in considering $t_c$ and $t_d$ (Gupta, 2001)

1. When $t_c = t_d$, the recession limb begins at $t = t_c$. Up to $t_c$, discharge is evaluated by Equation 5-80. The recession portion of the hydrograph is found using Equation 5-81, substituting $t_c$ for $t_d$.

2. When $t_c < t_d$, the discharge will initially rise according to Equation 5-80 up to $t_c$. Between $t_c$ and $t_d$, flow will remain constant at the outlet, and beyond $t_d$, Equation 5-81 applies.

3. When $t_c > t_d$, discharge is evaluated by Equation 5-80 up to $t = t_d$. Thereafter, it will remain constant from $t_d$ to $t_k$, expressed as

$$t_k = t_c + \frac{t_d}{m}\left[\frac{t_c}{t_d}\right] - 1 \quad (5-82)$$

After $t_k$, the recession limb is computed by Equation 5-81.

Applications in which rainfall intensity is not constant or when flow enters a well-defined channel, an analytical solution to the kinematic wave equation is not available and a numerical solution is required. In these cases, the corresponding model can be expressed as

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_L \quad (5-83)$$

and

$$Q = \alpha_q A^m \quad (5-84)$$

where $x$ is longitudinal distance, and $q_L$ is lateral inflow per unit length of channel. Thus, the kinematic wave equation can be expressed as

$$\frac{\partial Q}{\partial x} + \frac{1}{c_k} \frac{\partial Q}{\partial t} = q_L \quad (5-85)$$
or in terms of flow area,

\[ c_k \frac{\partial A}{\partial x} + \frac{\partial A}{\partial t} = q_L \]  \hspace{1cm} (5-86)

where the wave celerity \( c_k \) is defined as

\[ c_k = \alpha_m A^{n-1} = \frac{\partial Q}{\partial A} \]  \hspace{1cm} (5-87)

Because of the limits with respect to flow attenuation, the application of the kinematic wave equation works best for short, well-defined channel reaches, such as those typically found in urban drainage systems. Additional detail regarding numerical solution of the kinematic wave equation can be found in Chapter 6.

### 5.5 SOLVED PROBLEMS

#### Problem 5.1 Time of concentration (kinematic wave)

Consider a 20-acre urban drainage basin that consists predominantly (i.e., \( \cong 75 \) percent) of asphalt pavement. The average overland flow path is characterized as having a length and slope of 1,200 ft and 0.016 ft/ft, respectively. If rainfall intensity is described as

\[ i = \frac{1.75}{(0.285 + t_d)} \text{ in/hr}, \]

where \( t_d \) is the duration in hours, estimate the time of concentration for the basin.

**Solution**

Assuming \( n = 0.013 \) and \( i = 3.00 \text{ in/hr} \), Equation 5-13 can be written as

\[ t_c = \frac{0.938(1200)^{0.6}(0.013)^{0.6}}{(3.00)^{0.4}(0.016)^{0.3}} = 10.86 \text{ min} \]

Equating the time of concentration and the storm duration, the corresponding rainfall intensity is

\[ i = \frac{1.75}{(0.285 + \frac{10.36}{60})} = 3.82 \text{ in/hr} \]
The value of \( t_c \) is recomputed based on the newly-computed rainfall intensity, and the process is repeated until values of \( i \) converge. The following summarizes the solution, which yields a time of concentration of 9.77 min.

<table>
<thead>
<tr>
<th>Assumed ( i ) (in/hr)</th>
<th>( t_i ) (min)</th>
<th>Computed ( i ) (in/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>10.86</td>
<td>3.82</td>
</tr>
<tr>
<td>3.82</td>
<td>9.86</td>
<td>3.89</td>
</tr>
<tr>
<td>3.89</td>
<td>9.79</td>
<td>3.90</td>
</tr>
<tr>
<td>3.90</td>
<td>9.78</td>
<td>3.91</td>
</tr>
<tr>
<td>3.91</td>
<td>9.77</td>
<td>3.91 (OK)</td>
</tr>
</tbody>
</table>

This result should be viewed in light of the fact that the given length exceeds practical upper limits (i.e., 100 to 300 ft) on the kinematic wave equation. A means of overcoming this limitation is to divide the primary overland flow path into shorter segments and sum the corresponding times of concentration.

**Problem 5.2 Time of concentration (NRCS method)**

Solve Problem 5.1 using the NRCS lag equation for computing time of concentration.

**Solution**

Assuming \( CN = 98 \), the NRCS lag equation can be expressed as

\[
\begin{align*}
t_c &= \frac{(1200)^{0.8} \left[ \left( \frac{1000}{98} \right) - 9 \right]^{0.7}}{190(0.016)^{1/2}} \\
&= 13.77 \text{ min}
\end{align*}
\]

This result can be adjusted by factor \( M \) (i.e., Equation 5-16). For 75 percent imperviousness,

\[
M = (1 - 0.75) \left[ -6.8 \times 10^{-3} + \left( 3.4 \times 10^{-4} \right) \left( \frac{98}{98} \right) - \left( 4.3 \times 10^{-7} \right) \left( \frac{98}{98} \right)^2 - \left( 2.2 \times 10^{-8} \right) \left( \frac{98}{98} \right)^3 \right] = 0.874
\]

Therefore, \( t_c \) is 13.77 \times 0.874, or 12.00 min.
Problem 5.3  Time of concentration (upland method)

Solve Problem 5.1 using the NRCS upland method.

Solution

For a paved area and $S = 1.6$ percent, Figure 5-1 yields a velocity of approximately 2.5 fps. Alternatively, from Table 5-2, $K = 20.4$, and velocity can be computed as

$$V = 20.4\sqrt{0.016} = 2.6 \text{ fps}$$

Using the latter value, time of concentration can be computed by

$$t_c = \frac{1200}{60(2.6)} = 7.69 \text{ min}$$

Problem 5.4  Time of concentration (Kirpich)

Solve Problem 5.1 using the Kirpich equation.

Solution

Equation 5-20 can be written as

$$t_c = 0.0078\left(\frac{1200}{0.016}\right)^{0.77} = 9.00 \text{ min}$$

For an asphalt surface, Rossmiller (1980) recommends an adjustment factor of 0.4, which yields $t_c = 9 \times 0.4$ or 3.60 min.

Problem 5.5  Time of concentration (Izzard)

Solve Problem 5.1 using the Izzard equation for computing time of concentration.

Solution

Assuming a duration of 10 minutes, consistent with that determined using the kinematic wave equation,
\[ i = \frac{1.75}{0.285 + \frac{10}{60}} = 3.87 \text{ in/hr} \]

Note that \( iL = (3.87)(1200) = 4644 \), which is greater than the value of 500 recommended for use of the equation. Therefore, Izzard’s equation does not apply to this basin/storm event.

**Problem 5.6 Time of concentration (Kerby)**

Solve Problem 5.1 using the Kerby equation for computing time of concentration.

*Solution*

Assuming \( n = 0.013 \), Equation 5-22 can be written as

\[ t_c = \frac{0.83(0.013 \times 1200)^{0.47}}{0.016} = 23.87 \text{ min} \]

Note that general limits of the Kerby equation are exceeded in this problem (i.e., \( L > 1000 \text{ ft} \), \( S > 0.01 \text{ ft/ft} \), and \( A > 10 \text{ acres} \)); thus, results should be questioned.

**Problem 5.7 Time of concentration (FAA)**

Solve Problem 5.1 using the FAA method.

*Solution*

Assuming \( C = 0.9 \), Equation 5-23 is expressed as

\[ t_c = \frac{0.39(1.1 - 0.9)(1200)^{0.12}}{(0.016)^{0.13}} = 10.72 \text{ min} \]

**Problem 5.8 Time of concentration (Yen and Chow)**

Solve Problem 5.1 using the Yen and Chow equation for computing time of concentration.

*Solution*

Assuming \( K_Y = 0.7 \) and \( N = 0.012 \),
Comments: Based on results from Problems 5.1 through 5.8, time of concentration values range from 3.6 to 23.87 minutes, depending on the method of computation. A more realistic range based on limits of methods used is 9 to 12 minutes. This variability, however, does demonstrate the importance of critically evaluating results prior to use in design applications.

Problem 5.9 Time of concentration (Kirpich)

Assume that the watershed described in Problem 5.1 drains into a rectangular, concrete channel that is 200 ft long. If the 1-ft wide channel is laid on a slope of 0.5 percent and flows at a depth of 1 ft, evaluate the time of concentration using the Kirpich method.

Solution

From Problem 5.4, the overland flow \( t_c \) is 3.60 minutes. For the channel, assuming an adjustment factor of 0.2,

\[
t_c = 0.2 \times \left( \frac{200}{0.005} \right)^{0.77} \left( \frac{0.012}{0.016} \right)^{0.6} = 0.71 \text{ min}
\]

The total time of concentration is 3.60 + 0.71 or 4.31 min.

Problem 5.10 Time of concentration (upland method)

Solve Problem 5.9 using the NRCS upland method.

Solution

From Problem 5.3, the overland flow \( t_c \) is 7.69 minutes. For the rectangular channel, assuming \( n = 0.013 \),

\[
V = K \sqrt{S} = \left( \frac{K_m R^{2/3}}{n} \right) \sqrt{S} = \frac{K_m}{n} \left( \frac{B y}{B + 2 y} \right)^{2/3} \sqrt{S} = 1.49 \left( \frac{B y}{B + 2 y} \right)^{2/3} \sqrt{0.005} = 3.90 \text{ fps}
\]
Subsequent application of Equation 5-17 yields

\[ t_c = \frac{200}{60(3.90)} = 0.85 \text{ min} \]

The total time of concentration is 7.69 + 0.85 or 8.54 min.

**Problem 5.11 Time of concentration (Carter)**

Solve Problem 5.9 using the kinematic wave equation for overland flow and the Carter equation for channel flow.

**Solution**

Application of the kinematic wave equation yields an overland flow time of concentration of 9.77 minutes. Application of Equation 5-28 yields

\[ t_c = 100 \left( \frac{200 \times \frac{1}{5280 \text{ ft/mi}}}{0.005 \times \frac{5280 \text{ ft/mi}}{0.02}} \right)^{0.6} = 5.25 \text{ min} \]

Therefore, total time of concentration is 9.77 + 5.25 or 15.02 min.

**Problem 5.12 Time of concentration (upland method)**

Assume that the parabolic channel section shown in Figure P5-12 flows at an average depth, \( y \), of 0.5 m. The Manning roughness and slope of the channel are 0.015 and 0.02 m/m, respectively. For a longitudinal length of 100 m, evaluate the travel time associated with flow through the channel.

**Figure P5-12**

**Solution**

Using the geometric properties in Table 5-5 with \( b = 2.0 \text{ m}, y = 0.5 \text{ m}, \) and \( Y = 1.0 \text{ m}, \)
Substituting expressions for $R$ into Equations 5-25 and 5-19 yields,

$$V = K\sqrt{S} = \left(\frac{K_m R^{2/3}}{n}\right)\sqrt{S} = \frac{1.0}{0.015} \left(\frac{0.263}{0.02}\right)^{2/3} \sqrt{0.02} = 3.87 \text{ fps}$$

Then, assuming $t_c \equiv$ travel time,

$$t_c = \frac{200}{60(3.87)} = 0.86 \text{ min}$$

**Problem 5.13  Time of concentration (upland method)**

Evaluate the travel time associated with a 24-in diameter, 300-ft long cast iron pipe flowing at a depth of 5.5 inches and laid on a slope of 0.004 ft/ft.

**Solution**

Assuming $n = 0.013$ and using Table 5-5,

$$\theta = 2 \cos^{-1}\left(1 - \frac{2y}{D}\right) = 2 \cos^{-1}\left(1 - \frac{2(5.5)}{24}\right) = 2.0 \text{ rad}$$

$$A = \frac{(\theta - \sin \theta)D^2}{8} = \frac{(2.0 - \sin 2.0)(2)^2}{8} = 0.545 \text{ ft}^2$$

$$R = \frac{D}{4} \left(1 - \frac{\sin \theta}{\theta}\right) = \frac{2}{4} \left(1 - \frac{\sin 2.0}{2.0}\right) = 0.273 \text{ ft}$$

Flow velocity is computed by
\[ V = \frac{K \sqrt{S}}{n} \left( \frac{K_m R^{2/3}}{n} \right) \sqrt{S} = \frac{1.49}{0.013} (0.273)^{2/3} \sqrt{0.004} = 3.05 \text{ fps} \]

and the travel time (i.e., time of concentration) is

\[ t_c = \frac{300}{60(3.05)} = 1.64 \text{ min} \]

Note that the corresponding discharge is evaluated by the Manning equation (i.e., Equation 5-3), or alternatively

\[ Q = VA = (3.05)(0.545) = 1.66 \text{ cfs} \]

**Problem 5.14 Rational method**

A new 7-ac suburban development is to be drained by a storm sewer that connects to a municipal drainage system. The time of concentration for the basin is 15 minutes, and the local IDF relationship can be approximated as

\[ i = 5.6 - 0.2t_d \]

where \( t_d \) is rainfall duration in hours. Estimate the peak runoff using the rational method.

**Solution**

Rainfall intensity can be computed using the given IDF relationship

\[ i = 5.6 - 0.2 \left( \frac{15}{60} \right) = 5.55 \text{ in/hr} \]

Assuming a runoff coefficient of 0.4, peak discharge is computed using Equation 5-29, written as

\[ Q_p = \frac{(0.4)(5.55)(7)}{1.0} = 15.5 \text{ cfs} \]

**Problem 5.15 Rational method**

Evaluate how peak flow changes if the development described in Problem 5.14 consists of two smaller subbasins: one has a drainage area of five acres and a runoff coefficient of 0.4, while the other drains two acres and has a runoff coefficient of 0.7. Compare the answer to that if the entire basin is characterized by a median runoff coefficient of 0.55.
Solution

The composite runoff coefficient for the two subbasins is computed from Equation 5-30, written as

\[ C_c = \frac{(0.4 \times 5) + (0.7 \times 2)}{7} = 0.49 \]

Peak discharge is computed by

\[ Q_p = \frac{(0.49)(5.55)(7)}{1.0} = 19.0 \text{ cfs} \]

which represents more a 20 percent increase in flow. If the median runoff coefficient \( C = 0.55 \) is used, \( Q_p \) rises to 21.3 cfs. This represents a 12 percent increase simply by averaging land use characteristics. The implication is that use of composite coefficients prevents unnecessary oversizing of drainage facilities.

Problem 5.16 TR-55 method

A 3.0-km² watershed having 0.5 percent pond/swamp area has a curve number of 80, and \( t_c \) equal to 1.6 hours. Estimate the peak discharge for a NRCS Type 1 24-hour rainfall of 15.2 cm (6.0 in).

Solution

From Chapter 3, potential maximum retention can be evaluated by

\[ S_r = \frac{1000}{CN} - 10 = \frac{1000}{80} - 10 = 2.5 \text{ in} = 6.35 \text{ cm} \]

For \( P_{24} = 15.2 \text{ cm} \), the depth of rainfall excess is

\[ P_e = \frac{(P_{24} - 0.2S_r)^2}{P_{24} + 0.8S_r} = \frac{[15.2 - 0.2(6.35)]^2}{15.2 + 0.8(6.35)} = 9.57 \text{ cm} \]

Interpolating from Table 5-7 for 0.5 percent pond/swamp area yields \( F_p = 0.91 \). Furthermore, by definition,

\[ \frac{I_a}{P_{24}} = \frac{0.2S_r}{P_{24}} = \frac{0.2(6.35)}{15.2} = 0.084 \]
Since, \( I_a/P_{24} < 0.1 \), values of \( C_0, C_1, \) and \( C_2 \) corresponding to \( I_a/P_{24} = 0.1 \) and a Type I rainfall are read from Table 5-8. These values are \( C_0 = 2.30550, C_1 = -0.51429, \) and \( C_2 = -0.11750 \). Then, for \( t_c = 1.6 \) hours, application of Equation 5-34 yields

\[
\log q_u = 2.30550 + (-0.51429)\log 1.6 + (-0.11750)(\log 1.6)^2 - 2.366
\]

Solving for the unit peak discharge yields \( q_u = 0.676 \text{ (m}^3/\text{s)/cm/km}^2 \). Finally, peak discharge is computed by Equation 5-32, expressed as

\[
Q_p = q_u A P F_p = (0.676)(3.0)(9.57)(0.91) = 17.66 \text{ m}^3/\text{s} = 623.7 \text{ cfs}
\]

**Problem 5.17 USGS regression equations**

Consider a 15-mi\(^2\) watershed having a slope of 30 ft/mi and a 2-year, 2-hour rainfall of 2.0 in. In its current state, watershed storage is 4 percent, \( DF \) equals 3, and 15 percent of the basin is impervious. Over the next 25 years, storage is expected to increase to 6 percent, \( DF \) will rise to 9, and 25 percent of the basin will be impervious. For a return period of 50 years, evaluate peak discharge for current and future conditions. Assume that \( RQ \) for the same return period is 3,500 cfs.

**Solution**

Under current conditions,

\[
UQ_{50} = 2.67(15)^{0.29}(30)^{0.15}(2 + 3)^{0.74}(4 + 8)^{-0.53}(13 - 3)^{0.28}15^{0.06}3500^{0.62}
= 4,181 \text{ cfs}
\]

In 25 years, discharge will increase to

\[
UQ_{50} = 2.67(15)^{0.29}(30)^{0.15}(2 + 3)^{0.74}(6 + 8)^{-0.53}(13 - 9)^{-0.28}25^{0.06}3500^{0.62}
= 5,135 \text{ cfs}
\]

which represents more than a 20 percent increase in peak flow.

**Problem 5.18 Convolution**

Given the rainfall excess and 1-hour unit hydrograph (UH) below, evaluate the corresponding direct runoff hydrograph. Assume a constant 0.3 in/hr rate of abstractions.
Time (hr)  |  1  |  2  |  3  |  4  |  5  |  6  |  7  |  8  \\
---|---|---|---|---|---|---|---|---
Intensity (in)  | 0.5 | 1.0 | 1.7 | 0.5 | -  | -  | -  | -  \\
UH (cfs/in)  | 100 | 320 | 450 | 370 | 250 | 160 | 90 | 40

Solution

The number of rainfall excess intervals, $M$, is equal to 4. Subtracting abstractions from total rainfall yields four 1-hour rainfall pulses as follows: $P_1 = 0.2$ in, $P_2 = 0.7$ in, $P_3 = 1.4$ in, and $P_4 = 0.2$ in. In addition, there are 8 unit hydrograph ordinates, so $N - M + 1 = 8$, and the number of direct runoff hydrograph ordinates, $N$, will be $8 + M - 1 = 11$. Applying Equation 5-43 to the first time interval, $n = 1$, runoff is evaluated as

$$Q_1 = P_1 U_1 = (0.2)(100) = 20\text{ cfs}$$

For the second and third intervals, $n = 2$ and 3,

$$Q_2 = P_2 U_2 + P_1 U_1 = (0.2)(320) + (0.7)(100) = 134\text{ cfs}$$

and

$$Q_3 = P_3 U_3 + P_2 U_2 + P_1 U_1 = (0.2)(450) + (0.7)(320) + (1.4)(100) = 454\text{ cfs}$$

Formulation of similar equations will continue until $n = N = 11$. Referring to the summary of computations in Table P5-18, note that Column 3 shows the direct runoff hydrograph resulting from $P_1 = 0.2$ in; Column 4 shows the direct runoff from $P_2 = 0.7$ in; etc. Column 7 shows the total direct runoff hydrograph from the cumulative rainfall event. Each ordinate in the column is equivalent to the sum of ordinates across each row.

Problem 5.19 Deconvolution

Given the excess rainfall distribution and resulting direct runoff hydrograph from Problem 5.18, apply deconvolution to evaluate the corresponding, original 1-hour unit hydrograph.

Solution

The number of rainfall excess intervals, $M$, is equal to 4, and the number of direct runoff ordinates, $N$, is eleven. Therefore, there will be 8 unit hydrograph ordinates (i.e., $N - M + 1 = 8$). From Equation 5-44, a total of eleven equations can be written in terms of 8 unknowns, as follows:
## Table P5-18

<table>
<thead>
<tr>
<th>Time, ( n ) (hr)</th>
<th>Rainfall (in)</th>
<th>Unit hydrograph ordinates (cfs/in)</th>
<th>Excess rainfall (in)</th>
<th>Direct runoff (cfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5 1.0 1.7 0.5</td>
<td>0.2 0.7 1.4 0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>20</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>320</td>
<td>64</td>
<td>70</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>450</td>
<td>90</td>
<td>224</td>
<td>140</td>
</tr>
<tr>
<td>4</td>
<td>370</td>
<td>74</td>
<td>315</td>
<td>448</td>
</tr>
<tr>
<td>5</td>
<td>250</td>
<td>50</td>
<td>259</td>
<td>630</td>
</tr>
<tr>
<td>6</td>
<td>160</td>
<td>32</td>
<td>175</td>
<td>518</td>
</tr>
<tr>
<td>7</td>
<td>90</td>
<td>18</td>
<td>112</td>
<td>350</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>8</td>
<td>63</td>
<td>224</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>28</td>
<td>126</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>56</td>
</tr>
<tr>
<td>11</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
Q_1 = P_1U_1 \\
Q_2 = P_2U_1 + P_1U_2 \\
\ldots \\
Q_4 = P_4U_1 + P_3U_2 + P_2U_3 + P_1U_4 \\
Q_5 = P_4U_2 + P_3U_3 + P_2U_4 + P_1U_5 \\
\ldots \\
Q_{10} = P_4U_7 + P_3U_8 \\
Q_{11} = P_4U_8 
\]

Only the first eight equations are needed to solve for the unknown unit hydrograph ordinates. As an example, consider \( n = 1 \) and 2,
\[ U_1 = \frac{Q_1}{P_1} = \frac{20}{0.2} = 100 \text{ cfs/in} \]

\[ U_2 = \frac{Q_2 - P_2U_1}{P_1} = \frac{134 - 0.7(100)}{0.2} = 320 \text{ cfs/in} \]

Remaining computations proceed in a similar manner. Table P5-19 provides a summary of computations and lists the unit hydrograph in Column 3.

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Direct runoff (cfs)</th>
<th>Unit hydrograph (cfs/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>134</td>
<td>320</td>
</tr>
<tr>
<td>3</td>
<td>454</td>
<td>450</td>
</tr>
<tr>
<td>4</td>
<td>857</td>
<td>370</td>
</tr>
<tr>
<td>5</td>
<td>1003</td>
<td>250</td>
</tr>
<tr>
<td>6</td>
<td>815</td>
<td>160</td>
</tr>
<tr>
<td>7</td>
<td>554</td>
<td>90</td>
</tr>
<tr>
<td>8</td>
<td>345</td>
<td>40</td>
</tr>
<tr>
<td>9</td>
<td>186</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>74</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

**Problem 5.20 S-hydrograph**

Convert the 1-hour unit hydrograph provided in Problem 5.18 to a 4-hour unit hydrograph using the S-hydrograph method.

**Solution**

Table P5-20 summarizes the evaluation of the new unit hydrograph. Specific entries in the table are as follows:

*Columns 1 and 2*: The current 1-hour unit hydrograph

*Column 3*: Represents an indefinite series of unit hydrographs, each lagged by the current duration; for illustrative purposes, only three unit hydrographs are shown.
Column 4: 1-hour S-curve found by summing values in Columns 2 and 3
Column 5: S-curve from Column 4 lagged by the new duration of 4 hours
Column 6: Difference between the current and lagged S-curves
Column 7: The 4-hour unit hydrograph is computed by multiplying the values in Column 6 by the ratio of the current duration to the desired duration, or 0.25

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>1-hr UH (cfs/in)</th>
<th>Lagged 1-hr UH (cfs/in)</th>
<th>1-hr S-curve (cfs/in)</th>
<th>Lagged S-curve (cfs/in)</th>
<th>Difference (cfs/in)</th>
<th>4-hr UH (cfs/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>0</td>
<td>100</td>
<td>-</td>
<td>100</td>
<td>105</td>
</tr>
<tr>
<td>2</td>
<td>320</td>
<td>100</td>
<td>420</td>
<td>-</td>
<td>420</td>
<td>105</td>
</tr>
<tr>
<td>3</td>
<td>450</td>
<td>320</td>
<td>870</td>
<td>-</td>
<td>870</td>
<td>218</td>
</tr>
<tr>
<td>4</td>
<td>370</td>
<td>450</td>
<td>1240</td>
<td>0</td>
<td>1240</td>
<td>310</td>
</tr>
<tr>
<td>5</td>
<td>250</td>
<td>370</td>
<td>1490</td>
<td>100</td>
<td>1390</td>
<td>348</td>
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<td>-</td>
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</tr>
</tbody>
</table>

**Problem 5.21 NRCS dimensionless unit hydrograph**

Derive the NRCS unit hydrograph for a 10-mi² watershed having an average slope of 0.025 ft/ft. The hydraulic flow length from the catchment boundary to the outlet is 2.5 mi (13,200 ft), and the basin is characterized by a curve number of 80.

**Solution**

Time of concentration, $t_c$, is computed using the NRCS lag equation, or
Time-to-peak discharge, $t_p$, unit hydrograph base time, $t_b$, peak discharge, $Q_p$, and effective duration, $t_d$, are computed as follows:

\[
t_p = \frac{2}{3}(2.6) = 1.7 \text{ hrs}
\]

\[
t_b = 4.0(t_p) + t_p = 5(1.7) = 8.5 \text{ hrs}
\]

\[
Q_p = \frac{484(10)}{1.7} = 2847 \text{ cfs/in}
\]

\[
t_d = 0.133(2.6) = 0.35 \text{ hrs}
\]

The 0.35-hour unit hydrograph, as shown in Figure P5-21, can be derived by multiplying values in Table 5-9 by respective values of $t_p$ and $Q_p$.

**Problem 5.22 NRCS triangular unit hydrograph**

For the basin described in Problem 5.21, evaluate the NRCS triangular unit hydrograph.

*Solution*

Using results from Problem 5.21, the base time for the triangular unit hydrograph is

\[
t_b = 1.67(t_p) + t_p = 2.67(1.7) = 4.5 \text{ hrs}
\]

Thus, the 0.35-hour, triangular unit hydrograph is made by plotting points (0 hrs, 0 cfs/in), (1.7 hrs, 2847 cfs/in) and (4.5 hrs, 0 cfs/in), as shown in Figure P5-22.

**Problem 5.23 Snyder unit hydrograph**

Using Snyder’s method, derive a 1-hour synthetic unit hydrograph for the basin described in Problem 5.21. Assume that $L_{CA} = 1$ mi, $C_i = 1.9$, and $C_p = 0.6$. 
Solution

Lag time is computed using Equation 5-48.
From Equation 5-49, the duration of rainfall excess is

\[ t_d = \frac{2.5}{5.5} = 0.45 \text{ hrs} \]

However, the desired duration, \( t_{da} \), is 1 hour, so the lag time should be adjusted according to Equation 5-50.

\[ t_{la} = 2.5 + 0.25(1.0 - 0.45) = 2.64 \text{ hrs} \]

Equations 5-51 and 5-52 can be used to determine the time-to-peak discharge, \( t_p \), and peak discharge, \( Q_p \).

\[ t_p = 2.64 + 0.5(1.0) = 3.14 \text{ hrs} \]

\[ Q_p = \frac{640(0.6)(10)}{2.64} = 1455 \text{ cfs/in} \]

The unit hydrograph widths at discharges equal to 50 and 75 percent of the peak discharge are

\[ W_{50} = 770\left(\frac{10}{1455}\right)^{1.08} = 3.55 \text{ hrs} \]

and

\[ W_{75} = 440\left(\frac{10}{1455}\right)^{1.08} = 2.03 \text{ hrs} \]

The unit hydrograph base time, \( t_b \), is computed by finding that which guarantees an area under the curve equivalent to 1 inch of rainfall excess. For \( A \) in \( \text{m}^2 \), \( Q_p \) in \( \text{cfs} \), and \( W_{50} \) and \( W_{75} \) in hours,

\[
1 \text{ in} = \left[ \left( \frac{t_b + W_{50}}{2} \right) \left( \frac{Q_p}{2} \right) + \left( \frac{W_{75} + W_{50}}{2} \right) \left( \frac{Q_p}{4} \right) + \left( \frac{W_{75}}{2} \right) \left( \frac{Q_p}{4} \right) \right] (\text{hr - cfs}) \times \\
\left[ \left( \frac{1}{A} \right) \left( \frac{1 \text{ mi}^2}{(5280)^2 \text{ ft}^2} \right) \left( \frac{12 \text{ in}}{1 \text{ ft}} \right) \left( \frac{3600 \text{ sec}}{1 \text{ hr}} \right) \right]
\]

Solving for \( t_b \) yields
A total of seven coordinates are now known and can be used to define the resulting unit hydrograph shown in Figure P5-23. From the starting point of (0,0) (i.e., A), the remaining points are as follows:

\[ B \rightarrow \left(t_p - \frac{W_{50}}{3}, 0.5Q_p\right) = (1.96, 728) \]

\[ C \rightarrow \left(t_p - \frac{W_{75}}{3}, 0.75Q_p\right) = (2.46, 1091) \]

\[ D \rightarrow \left(t_p, Q_p\right) = (3.14, 1455) \]

\[ E \rightarrow \left(t_p + \frac{2}{3}W_{75}, 0.75Q_p\right) = (4.49, 1091) \]

\[ F \rightarrow \left(t_p + \frac{2}{3}W_{50}, 0.5Q_p\right) = (5.51, 728) \]

\[ G \rightarrow (t_b, 0) = (10.4, 0) \]
Problem 5.24 Clark unit hydrograph

Use Clark’s method to develop a 1-hour unit hydrograph for the watershed described in Problem 5.21. Assume $t_c = 3$ hours and a storage coefficient of $0.6t_c$.

Solution

For an assumed time increment, $\Delta t$, of 0.5 hours, Table P5-24 summarizes application of Clark’s method; specific entries are explained as follows:

Column 1: Time intervals from the beginning of excess rainfall

Column 2: Cumulative watershed area that contributes flow to the outlet at time $t$, obtained using Equation 5-55. For example, at $t = 1$ hour,

\[
A_c = 10 \times 640 \frac{mi^2}{ac} \times 1.414 \left(\frac{1}{3}\right)^{1.3} = 1741.6 \text{ ac}
\]

Column 3: Area of the watershed, in acres, that starts contributing flow to the outlet within the current time interval. This area can be plotted over time to produce the time area histogram, shown in Figure P5-24a.

Column 4: Inflow generated, obtained by multiplying values in Column 3 by 1-inch of rainfall excess, and dividing the result by $\Delta t$. For example, inflow at $t = 1$ hour is $1125.9 \text{ ac} \times 1 \text{ in} / 0.5 \text{ hrs}$. Note that 1 ac-in/hr = 1 cfs.

![Figure P5-24a](image-url)
**Column 5:** Outflow ordinates are obtained by routing the inflow hydrograph through a linear reservoir using Equation 5-58. The storage coefficient, $K_s$, is $0.6t_c = 1.8$ hours. Then,

\[
C_1 = \frac{0.5}{1.8 + 0.5 \times 0.5} = 0.244
\]

\[
C_2 = 1 - 0.244 = 0.756
\]

Therefore,

\[
Q_t = 0.244I_t + 0.756Q_{t-1}
\]

For example, at $t = 1$ hour,

\[
Q_t = (0.244)(2251.8) + (0.756)(300.5) = 776.6 \text{ cfs/in}
\]

As previously discussed, since runoff ordinates will theoretically continue for an infinite duration, it is customary to truncate the hydrograph when discharge volume exceeds 0.995 inches. As such, only the first twenty ordinates are given in Table P5-24.

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>$A_{c,t}$ (ac)</th>
<th>$\Delta A$ (ac)</th>
<th>$I_t$ (cfs/in)</th>
<th>$Q_t$ (cfs/in)</th>
<th>$Q_{I,H}$ (cfs/in)</th>
<th>Lagged $Q_{I,H}$ (cfs/in)</th>
<th>$Q_{1-hr}$ (cfs/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>615.7</td>
<td>615.7</td>
<td>1231.4</td>
<td>300.5</td>
<td>150.3</td>
<td>0</td>
<td>75.2</td>
</tr>
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<td>1125.9</td>
<td>2251.8</td>
<td>776.6</td>
<td>538.6</td>
<td>0</td>
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</tr>
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<td>2251.8</td>
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<td>33.6</td>
<td>39.0</td>
<td>68.2</td>
<td>53.6</td>
</tr>
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</table>
Column 6: Clark’s instantaneous unit hydrograph ordinates obtained by averaging the values in Column 5 over the computational time step (i.e., application of Equation 5-61).

Column 7: Represents Clark’s unit hydrograph lagged by the desired duration, equal to one hour.

Column 8: Ordinates of the 1-hour synthetic unit hydrograph, shown in Figure 5-24b, obtained by averaging ordinates of the original and lagged hydrographs. For example, the ordinate value of the resulting unit hydrograph at $t = 1$ is the average 538.6 and 0 cfs, or 269.3 cfs.

![Figure P5-24b](image)

**Problem 5.25 Tri-triangular unit hydrograph**

Use the tri-triangular unit hydrograph method to derive a 1-hour synthetic unit hydrograph for an 10-mi$^2$ watershed. Assume that $\forall_1$, $\forall_2$, and $\forall_3$ are 30, 50, and 20 percent, respectively; $t_1$, $t_2$, and $t_3$ are 1, 4, and 12 hours, respectively, and $K_1$, $K_2$, and $K_3$ are 2, 3, and 3.

**Solution**

Assuming that 1 inch of rainfall excess is collected over the 1-hour duration, the total volume of runoff generated from the watershed is the product of depth and basin area, expressed as

$$\left(\frac{1}{12}\right)(10)(5280)^2 = 23,232,000 \text{ ft}^3$$
The volume of direct runoff allocated to the first triangle (i.e., the triangle representing fastest responding components of the watershed) is

\[(\forall) (23,232,000) = (0.3)(23,232,000) = 6,969,600 \text{ ft}^3\]

The volumes associated with the second and third triangles, computed in a similar manner, are 11,616,000 and 4,646,400 ft$^3$, respectively. The base time for the first triangle is

\[t_{b1} = t_1 + t_1K_1 = 1 + (1)(2) = 3 \text{ hrs}\]

Similarly, the base times for the second and third triangles are \(t_{b2} = 16\) hours and \(t_{b3} = 48\) hours. Finally, the peak flow for each triangle is computed as

\[Q_{p1} = \frac{2 \times \forall}{T_{b1}} = \frac{2 \times 6,969,600}{3 \times 60 \times 60} = 1,290.6 \text{ cfs/in}\]

\[Q_{p2} = \frac{2 \times \forall}{T_{b2}} = \frac{2 \times 11,616,000}{16 \times 60 \times 60} = 403.3 \text{ cfs/in}\]

\[Q_{p3} = \frac{2 \times \forall}{T_{b3}} = \frac{2 \times 4,646,400}{48 \times 60 \times 60} = 53.8 \text{ cfs/in}\]

With the times to peak discharge, base times, and peak flows of each triangle, the resulting 1-hour unit hydrograph, shown in Figure P5-25, is generated by aggregating ordinates of the three triangles at any time \(t\).
Problem 5.26 Espey-Altman unit hydrograph

Use the Espey-Altman method to derive a 10-minute synthetic unit hydrograph for a 10-mi² watershed. The length and slope of the primary channel is 2.5 mi (13,200 ft) and 0.025 ft/ft, respectively. Assume the basin is 30 percent impervious and that φ = 0.8.

Solution

Time-to-peak discharge and peak flow are computed from Equations 5-62 and 5-63, expressed as

\[ t_p = 3.1 \frac{L^{0.23} \phi^{0.57}}{S^{0.23} \rho^{0.18}} = 3.1 \frac{(13200)^{0.23} (0.8)^{0.57}}{(0.025)^{0.23} (30)^{0.18}} = 26.4 \text{ min} = 0.44 \text{ hr} \]

and

\[ Q_p = 31.62 \times 10^3 \frac{A^{0.96}}{t_p^{0.07}} = 31.62 \times 10^3 \left( \frac{10}{26.4} \right)^{0.07} = 8687 \text{ cfs/in} \]

The shape of the hydrograph is defined by base time, \( W_{50} \), and \( W_{75} \), or

\[ t_b = 125.89 \times 10^3 \frac{A}{Q_p^{0.93}} = 125.89 \times 10^3 \left( \frac{10}{8687} \right)^{0.93} = 228.1 \text{ min} = 3.80 \text{ hr} \]

\[ W_{50} = 16.22 \times 10^3 \frac{A^{0.93}}{Q_p^{0.92}} = 16.22 \times 10^3 \left( \frac{10}{8687} \right)^{0.92} = 32.8 \text{ min} = 0.55 \text{ hr} \]

\[ W_{25} = 3.24 \times 10^3 \frac{A^{0.79}}{Q_p^{0.78}} = 3.24 \times 10^3 \left( \frac{10}{8687} \right)^{0.78} = 16.9 \text{ min} = 0.28 \text{ hr} \]

Similar to the solution to Problem 5.23, these values yield a total of seven coordinates of the resulting unit hydrograph, shown in Figure P5-26.

Problem 5.27 Colorado urban hydrograph

Using the Colorado urban hydrograph procedure, develop the 1-hour unit hydrograph for a 0.25 mi² watershed having an average slope of 0.025 ft/ft. Assume that \( L_{Cd} = 0.2 \) mi, \( L = 0.4 \) mi, and that 30 percent of the basin is impervious.
Solution

Using coefficients from Table 5-10, application of Equations 5-70 and 5-71 for $C_t$ and $P_f$ yields

$$C_t = ap^2 + bp + c = (2.3 \times 10^{-5})(30)^2 - (0.00224)(30) + 0.146 = 0.0995$$

$$P_f = dp^2 + ep + f = (0.00245)(30)^2 - (0.012)(30) + 2.16 = 4.005$$

The peaking coefficient is computed by

$$C_p = P_f C_t A^{0.15} = (4.005)(0.0995)(0.25)^{0.15} = 0.32$$

From Equation 5-67, the lag time of the watershed is

$$t_L = C_t \left( \frac{LL_{CA}}{\sqrt{S}} \right)^{0.48} = 0.0995 \left( \frac{0.4(0.2)}{\sqrt{0.025}} \right)^{0.48} = 0.072 \text{ hrs}$$

and the time to peak is

$$t_p = 0.072 + (0.5)(1) = 0.572 \text{ hrs}$$
Peak flow is computed from Equation 5-68, expressed as

\[ Q_p = \frac{640C_pA}{t_p} = \frac{640(0.32)(0.25)}{0.572} = 89.5 \text{ cfs / in} \]

The widths of the unit hydrograph at 50 and 75 percent of the peak are computed from Equations 5-72 and 5-73, written as

\[ W_{50} = \frac{500}{\left( \frac{Q_p}{A} \right)} = \frac{500}{\left( \frac{89.5}{0.25} \right)} = 1.40 \text{ hrs} \]

\[ W_{75} = \frac{260}{\left( \frac{Q_p}{A} \right)} = \frac{260}{\left( \frac{89.5}{0.25} \right)} = 0.73 \text{ hrs} \]

Finally, \( W_{50} \) and the \( W_{75} \) must be distributed around the peak. For \( W_{50} \), the width to the left side of the peak flow is the smaller of 0.35\( W_{50} \) (i.e., 0.49 hours) and 0.6\( t_p \) (0.34 hours); thus the latter is chosen. Similarly, 0.65\( W_{50} \) (i.e., 0.91 hours) is assigned to the right-hand side. For \( W_{75} \), 0.424\( t_p \) (0.24 hours) is assigned to the left, and 0.55\( W_{75} \) (i.e., 0.40 hours) is assigned to the right. Finally, the base time of the unit hydrograph is determined so that the area under the curve corresponds to 1-inch of rainfall excess. As in Problem 5-23, for \( A \) in \( \text{mi}^2 \), \( Q_p \) in cfs, and \( W_{50} \) and \( W_{75} \) in hours, \( t_b \) can be evaluated by

\[ t_b = 2581 \frac{A}{Q_p} - 1.5W_{50} - W_{75} = 2581 \left( \frac{0.25}{89.5} \right) - 1.5(1.40) - 0.73 = 4.38 \text{ hrs} \]

The resulting unit hydrograph is illustrated in Figure P5-27.

**Problem 5.28 Delmarva unit hydrograph**

Develop a Delmarva unit hydrograph for the basin described in Problem 5.21.

**Solution**

Similar to NRCS unit hydrographs, time of concentration, \( t_c \), is computed using the NRCS lag equation, expressed as
Figure P5-27

Time-to-peak discharge, \( t_p \), and duration of effective rainfall, \( t_d \), are computed using same equations as that of the NRCS unit hydrographs.

\[
t_p = \frac{2}{3}(2.6) = 1.7 \text{ hrs}
\]

\[
t_d = 0.133(2.6) = 0.35 \text{ hrs}
\]

Unit hydrograph base time, \( t_b \), and peak discharge, \( Q_p \), are computed as follows:

\[
t_b = 9.0(t_p) + t_p = 10(1.7) = 17 \text{ hrs}
\]

\[
Q_p = \frac{284(10)}{1.7} = 1670.59 \text{ cfs/in}
\]

The 0.35-hour unit hydrograph is obtained by multiplying values in Table 5-11 by respective values of \( t_p \) and \( Q_p \), and is shown in Figure P5-28. Note that
volume of runoff under the unit hydrograph is equal to drainage area times an inch of rainfall depth.

\[
\text{Discharge (cfs)}
\]

Time (hrs)

\[
0 \quad 5 \quad 10 \quad 15 \quad 20
\]

\[
0 \quad 1200 \quad 800 \quad 400 \quad 0
\]

\[
2000
\]

Figure P5-28

Problem 5.29 Santa Barbara urban hydrograph

A 1-mi^2 watershed has a time of concentration equal to 55 minutes, and 40 percent of the basin is impervious. For the rainfall event described below, compute the direct runoff hydrograph.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rainfall intensity (in/hr)</td>
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<td>8</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Rainfall excess (in/hr)</td>
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<td>6</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Solution

Assuming a time increment of 10 minutes, instantaneous runoff can be computed from Equation 5-75, expressed as

\[
I = \left( \frac{5280 \text{ ft}}{\text{mi}} \right) \left[ i(0.4) + i_e(1.0 - 0.4) \right] \left( \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} \right)
\]

Runoff is evaluated by Equation 5-76, written as
\[ Q_t = Q_{t-1} + \left( \frac{10}{2(55)+10} \right) (I_{t-1} + I_t - 2Q_{t-1}) \]

Beginning with \( I_t = Q_t = 0 \), these two expressions are applied sequentially to obtain the direct runoff hydrograph. Table P5-29 summarizes the solution and lists the resulting hydrograph in Column 5. For illustrative purposes, discharge ordinates are shown only for 150 minutes after rainfall begins.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>( i ) (in/hr)</th>
<th>( i_c ) (in/hr)</th>
<th>( I_t ) (cfs)</th>
<th>( Q_t ) (cfs)</th>
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**Problem 5.30 San Diego modified rational hydrograph**

A 1-m² watershed has a time of concentration of one hour and runoff coefficient of 0.85. Assuming a 6-hour precipitation of two inches, derive a direct runoff hydrograph for the watershed using San Diego modified rational hydrograph.

**Solution**

The precipitation depth should be distributed across the 6-hour duration. Each rainfall block will have duration of the time of concentration. Therefore, the
number of rainfall blocks will be six (i.e., 6 hrs/1 hr).

Rainfall depth \( (P_0) \) for each of the blocks is determined using Equation 5-77 as

\[
P_1 = 0.124 \left( 1 - 0.6 \right) \left( 60 \right)^{0.355} \left( 1 - 0.6 \right) \left( 60 \right)^{0.355} = 1.01 \text{ in}
\]

\[
P_2 = 0.124 \left( 1 - 0.6 \right) \left( 120 \right)^{0.355} \left( 1 - 0.6 \right) \left( 60 \right)^{0.355} = 0.03 \text{ in}
\]

Likewise, \( P_3 = 0.21 \text{ in}; \ P_4 = 0.21 \text{ in}; \ P_5 = 0.14 \text{ in}; \) and \( P_6 = 0.13 \text{ in}. \)

The peak rainfall block (i.e., \( P_1 \)) would be placed at hour 4, and the other blocks are arranged according to the \((2/3, 1/3) \) rule as shown in Figure P5-30a.

For each rainfall block, peak flow is estimated using rational formula where rainfall intensity is determined for the blocks from the depths given above. For the peak rainfall block (i.e., \( P_1 \)), for example, peak runoff would be

\[
Q_1 = 0.85 \left( 1.01 \frac{\text{in}}{\text{hr}} \right) \left( 640 \text{ ac} \right) = 549 \text{ cfs}
\]

Following the same approach, peak flow corresponding to the other rainfall blocks would be \( Q_2 = 163 \text{ cfs}; \ Q_3 = 114 \text{ cfs}; \ Q_4 = 114 \text{ cfs}; \ Q_5 = 76 \text{ cfs}; \) and \( Q_6 = 70 \text{ cfs}. \)

\[ \text{Figure P5-30a} \]
A triangular unit hydrograph with peak ordinate equal to the peak flows computed above and time base of 2\( t_c \) is constructed for each rainfall block. The triangular hydrographs are then aggregated as shown in Figure P5-30b to generate the final direct runoff hydrograph.

![Figure P5-30b](image)

**Problem 5.31 Nonlinear reservoir model**

A 5-ac (217,800 ft\(^2\)) catchment has an average slope of 0.01 ft/ft and is characterized by a Manning roughness of 0.20. The basin is approximately 400-ft wide, and depression storage is estimated as 0.3 in. For the 20-min rainfall excess described below, evaluate the runoff from the basin.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0 – 5</th>
<th>5 - 10</th>
<th>10 - 15</th>
<th>15 - 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rainfall excess (in/hr)</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

**Solution**

With \( \Delta t \) equal to 5 minutes (300 seconds), Equation 5-79 can be written as

\[
y_2 = y_1 + \Delta t \left[ i_c - \frac{K_m B S^{1/2}}{A n} \left( \frac{y_1 + y_2}{2} - y_d \right)^{5/3} \right] = \\
y_2 + 300 \left[ i_c \left( \frac{1}{12} \times \frac{1}{60} \times \frac{1}{60} \right) - \frac{(1.49)(400)(0.01)^{1/2}}{(217800)(0.20)} \left( \frac{y_1 + y_2}{2} - \frac{0.3}{12} \right)^{5/3} \right]
\]
For given values of \( y_1 \), this expression is solved iteratively for \( y_2 = y \) over time. The corresponding runoff is then computed by Equation 5-78, written as

\[
Q = \frac{K_mB}{n} (y - y_d) \frac{5}{3} S_{1/2} = \left( \frac{1.49(400)}{0.20} \right) \left( y - \frac{0.3}{12} \right)^{5/3} (0.01)^{1/2}
\]

For example, for \( t = 5 \) to 10 minutes, \( y_1 = 0.05 \text{ ft} \), and \( i_e = 5 \text{ in/hr} \). \( y_2 \) is found to be 0.083 \( \text{ ft} \), and at this depth, application of Equation 5-76 yields \( Q = 2.590 \text{ cfs} \). Table P5-31 summarizes runoff results for the first hour. Beyond \( t = 60 \text{ minutes} \), discharge continues to gradually decrease, eventually reaching zero.

<table>
<thead>
<tr>
<th>Table P5-31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (min)</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>10</td>
</tr>
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<td>15</td>
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<td>20</td>
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<tr>
<td>55</td>
</tr>
<tr>
<td>60</td>
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</table>

**Problem 5.32 Kinematic wave model**

Rain falls at a rate of 4 cm/hr on a 100 m \( \times \) 100 m parking lot having a slope of 0.015 m/m and a Manning roughness of 0.025. Evaluate the resulting hydrograph if the rainfall duration is 5 minutes.
Solution

For a unit width,

\[
\alpha_s = \frac{K_m}{n} S^{1/2} = \frac{(t)}{0.025} (0.015)^{1/2} = 4.9
\]

and \( m = 5/3 \). Then, noting that 4 cm/hr is equivalent to \( 1.11 \times 10^{-5} \) m/s, time of concentration is computed from Equation 5-12, written as

\[
t_c = \left( \frac{L}{\alpha_s m-L} \right)^{1/m} = \left( \frac{100}{4.9(1.11 \times 10^{-5})^{0.667}} \right)^{0.6} = 586.9 \text{ sec} = 9.78 \text{ min}
\]

Since \( t_c > t_d \), the unit width discharge from \( t = 0 \) to \( t = 5 \) min is computed by Equation 5-80, expressed as

\[
q = \alpha (it)^m = 4.9 [1.11 \times 10^{-5} t]^{3/5}
\]

where \( t \) is in seconds. After 5 minutes, discharge will remain constant until

\[
t_k = 5 + \frac{5}{1.667} \left( \frac{9.78}{5} \right)^{1.667} - 1 = 11.18 \text{ min} = 670.7 \text{ sec}
\]

The recession limb is computed by

\[
L - \frac{q}{i} - m \alpha^{1/m} q^{m-1/m} (t-t_d) = 0 =
\]

\[
100 - \frac{q}{1.11 \times 10^{-5}} - 1.667 (4.9)^{0.6} q^{0.4} (t-300)
\]

which is solved iteratively for assumed values of \( t \). Table P5-32 lists the tabulated discharge results in Column 3, and Figure P5-32 illustrates the resulting hydrograph.
<p>| | | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td>Time (sec)</td>
<td>Time (min)</td>
<td>Discharge (m³/s/m × 10⁵)</td>
<td></td>
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<tr>
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<td>0</td>
<td>0</td>
<td></td>
</tr>
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Figure P5-32

REFERENCES CITED


